

Can we estimate and reduce major sources of forecast uncertainties employing a unified framework?

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Collaborators

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OUTLINE

- **Major sources of forecast uncertainty**
- **General (probabilistic) framework**
- **Experimental results**
- **Conclusions and future work**

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Major sources of forecast uncertainty

- Initial conditions
- Model errors (e.g., errors in dynamical equations, errors due to unresolved scales)
- Parametric errors (errors in empirical parameters)
- Forcing errors (e.g., errors in atmospheric forcing in hydrological models)
- Boundary conditions (e.g., lateral boundaries)

These uncertainties are not independent!

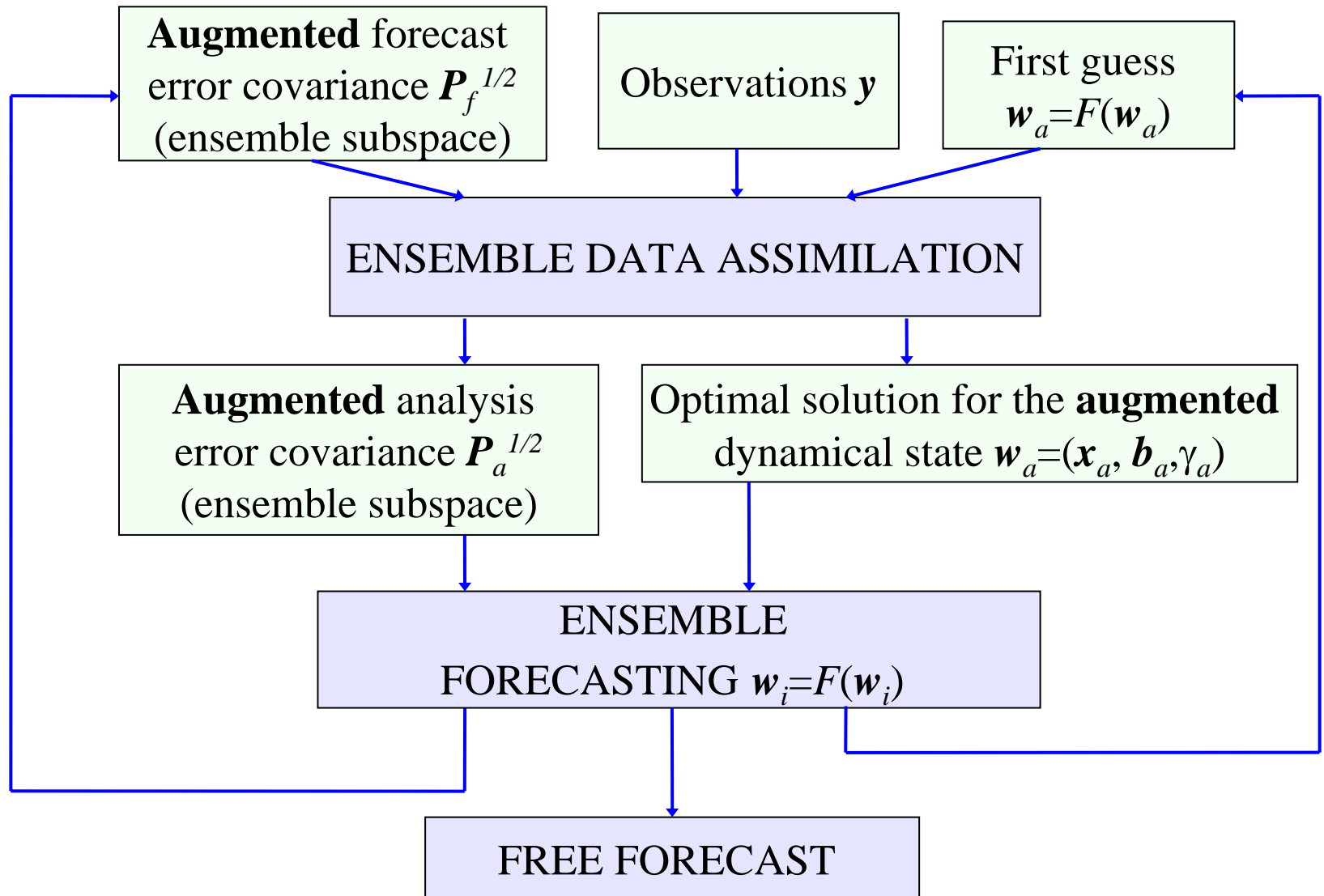


All sources of uncertainty should be taken into account *simultaneously* within a unified mathematical approach.



Verified analysis and forecast uncertainty

Can we do that?



Yes, it can be done employing ensemble data assimilation + state augmentation as a general probabilistic framework!

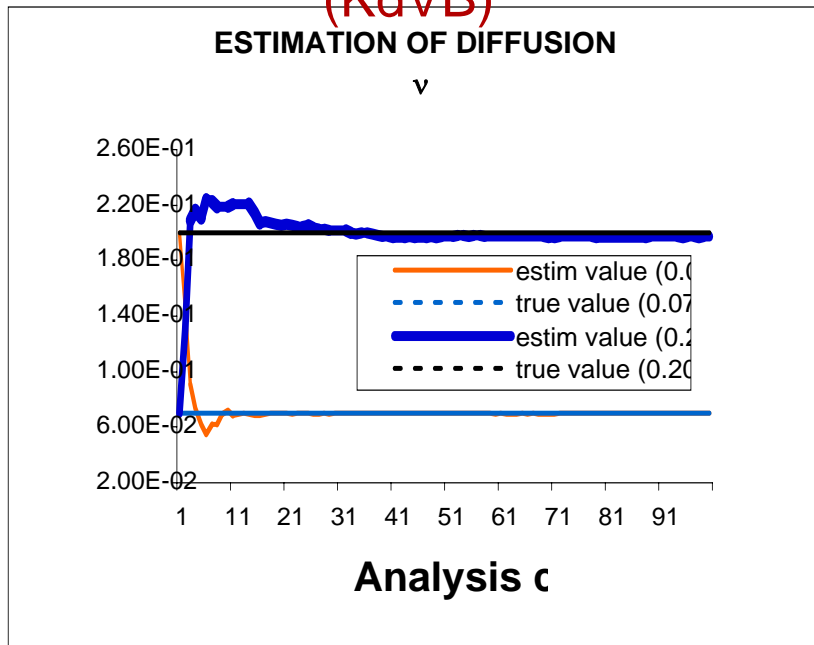
Research in this area

- Mitchell et al. 2000 (MWR)
- Anderson 2001 (MWR)
- Heemink et al. 2001 (MWR)
- Reichle et al. 2002 (MWR)
- Hansen 2002 (MWR)
- Houtekamer et al. 2005 (MWR)
- Hamill and Whitaker 2005 (MWR)
- Zupanski and Zupanski 2005 (MWR)

Parameter estimation

Korteweg-de Vries-Burgers model

(KdVB)

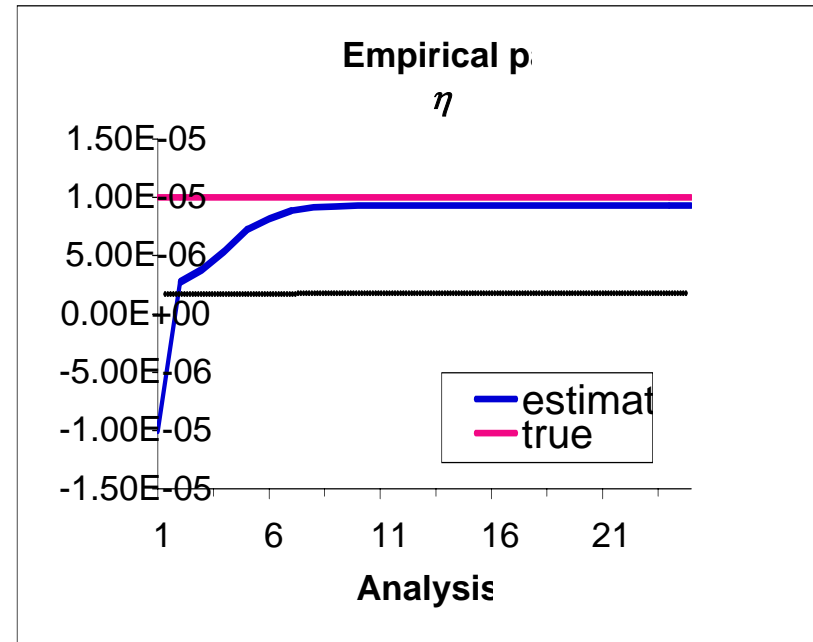


$$N_{state} = 101 + 1$$

$$N_{ens} = 102$$

$$N_{obs} = 101$$

RAMS (non-hydrostatic) model



$$N_{state} = 54000 + 1$$

$$N_{ens} = 50$$

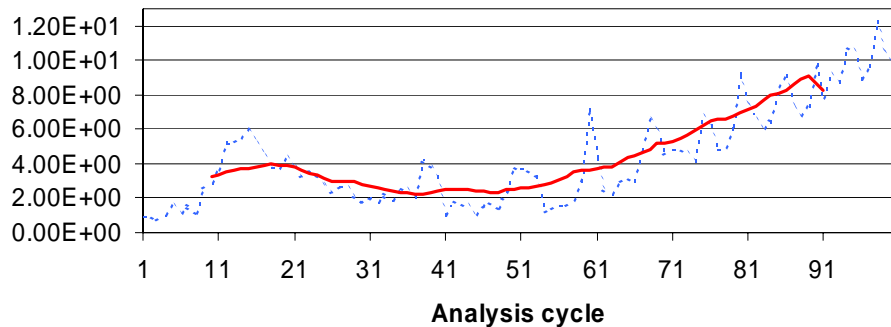
$$N_{obs} = 7200$$

The method is applicable to simple and complex models.

BIAS ESTIMATION, KdVB model

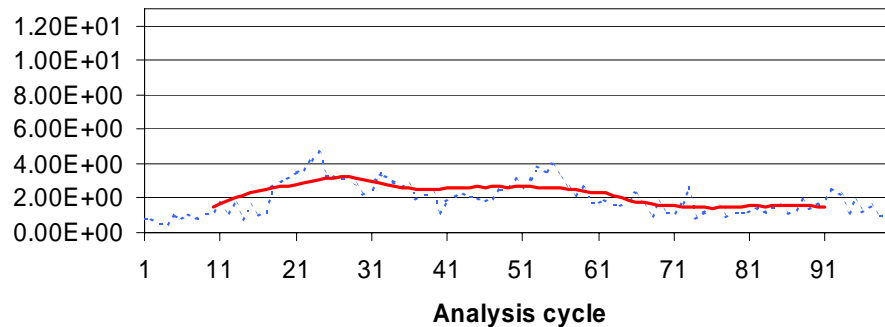
NEGLECT BIAS

INNOVATION χ^2 TEST (biased model)
(neglect_err, 10 ens, 10 obs)



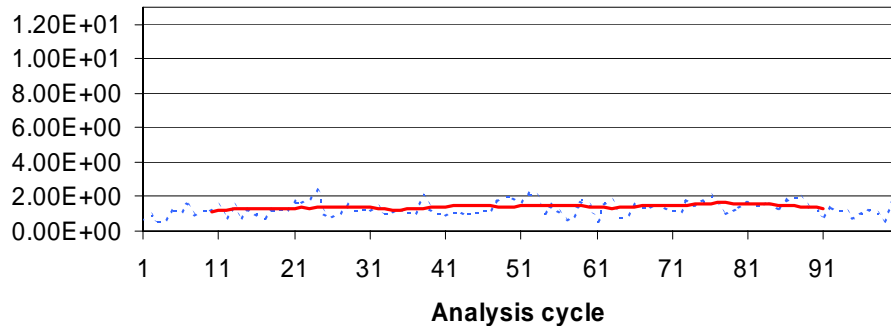
BIAS ESTIMATION (vector size=101)

INNOVATION χ^2 TEST (biased model)
(bias_estim, 10 ens, 10 obs, bias dim = 101)



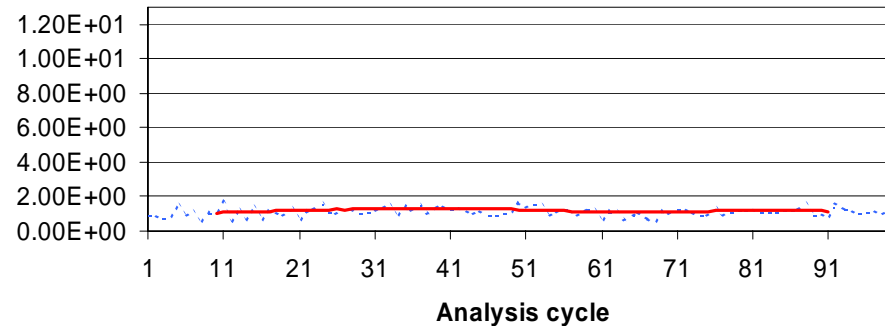
BIAS ESTIMATION (vector size=10)

INNOVATION χ^2 TEST (biased model)
(bias_estim, 10 ens, 10 obs, bias dim = 10)



NON-BIASED MODEL

INNOVATION χ^2 TEST (non-biased model)
(correct_model, 10 ens, 10 obs)



It is beneficial to reduce degrees of freedom of the model error.

How do we know the number of DOF?

Answer can be obtained by using the following 3 components within a general framework:

**Ensemble Data Assimilation +
State Augmentation +
Information theory**

Degrees of freedom (DOF) for signal (Rodgers 2000; Zupanski et al. 2005)

$$d_s = \text{tr}[(\mathbf{I} + \mathbf{C})^{-1}\mathbf{C}] = \sum_i \frac{\lambda_i^2}{(1 + \lambda_i^2)}$$

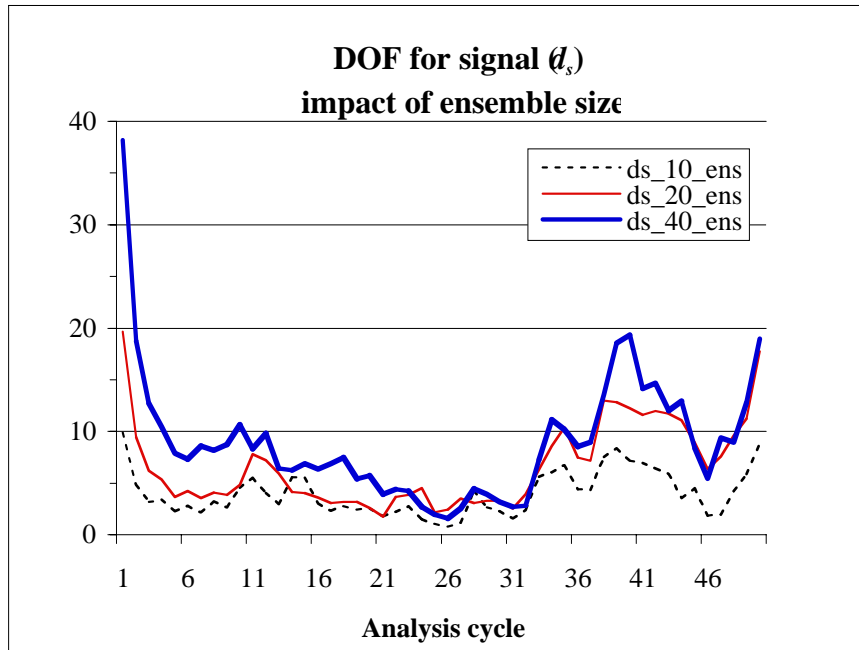
C - information matrix in ensemble subspace

**Shannon information content,
or entropy reduction**

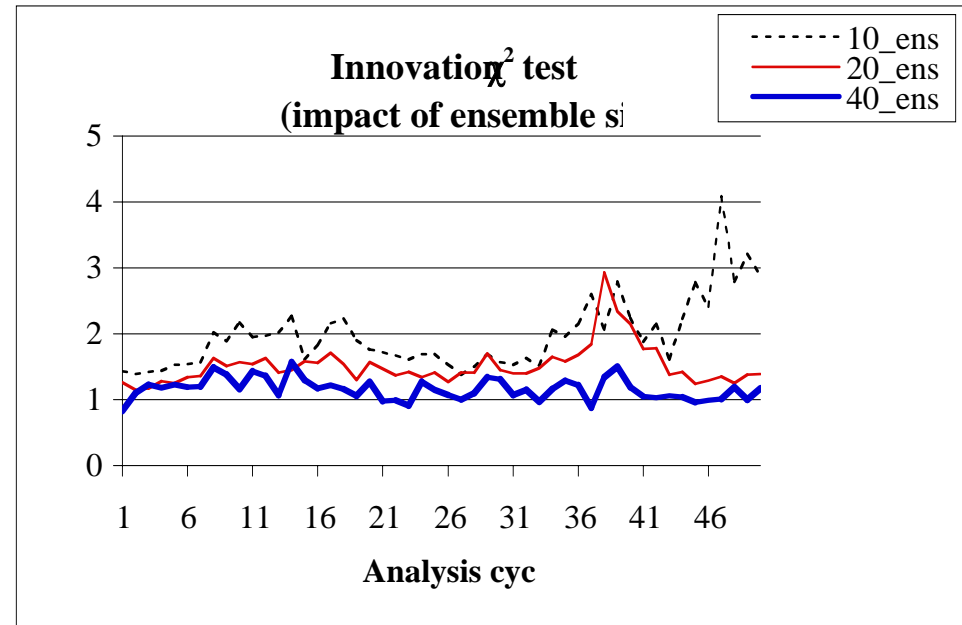
$$h = \frac{1}{2} \sum_i \ln(1 + \lambda_i^2)$$

Information content analysis GEOS-5 Single Column Model

DOF for Signal: 10, 20, and 20 Ens.



Chi-square statistics



$$N_{state} = 80; N_{obs} = 80$$

Chi-square test can be used to verify the results of the information content analysis.

CONCLUSIONS

- Unified methodologies for estimating all major sources of analysis and forecast uncertainties and for quantifying DOF of a dynamical system are desirable.
- Ensemble-based methods can address all these issues within the same mathematical (probabilistic) framework.
- Hydrological models are good candidates to test the general approach (for estimating multiple empirical parameters, unknown atmospheric forcing, and model bias).

FUTURE RESEARCH DIRECTIONS

- Evaluate this unified approach in application to complex dynamical models and real observations (e.g., atmospheric, hydrological, and carbon transport models).
- Estimate multiple components of the augmented state variable (e.g., initial conditions, multiple parameters, and model bias).
- Evaluate the unified approach employing non-Gaussian PDFs.
- Learn about model errors and possibly improve the models.

HOW?

$\mathbf{x}_n = M_{n,n-1}(\mathbf{x}_{n-1}, \mathbf{b}_{n-1}, \gamma_{n-1})$ - Dynamical model for standard model state \mathbf{x}

$\mathbf{b}_n = G_{n,n-1}(\mathbf{b}_{n-1})$ - Dynamical model for model error (bias) \mathbf{b}

$\gamma_n = S_{n,n-1}(\gamma_{n-1})$ - Dynamical model for empirical parameters γ

Define augmented state vector \mathbf{w}

$$\mathbf{w}_n = (\mathbf{x}_{n-1}, \mathbf{b}_{n-1}, \gamma_{n-1}) ,$$

And augmented dynamical model F

$$\mathbf{w}_n = F_{n,n-1}(\mathbf{w}_{n-1}) .$$



Find optimal solution (augmented analysis) \mathbf{w}_a by minimizing J (MLEF method):

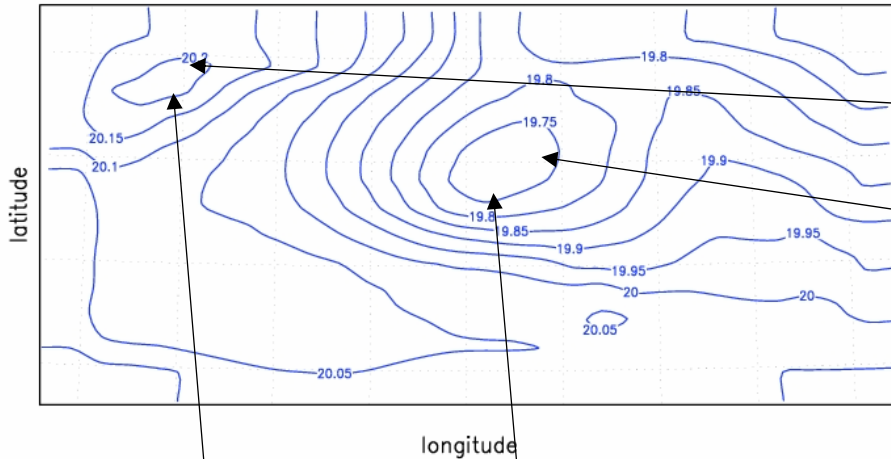
$$J = \frac{1}{2}[\mathbf{w} - \mathbf{w}_b]^T \mathbf{P}_f^{-1}[\mathbf{w} - \mathbf{w}_b] + \frac{1}{2}[H[F(\mathbf{w})] - \mathbf{y}_{obs}]^T \mathbf{R}^{-1}[H[F(\mathbf{w})] - \mathbf{y}_{obs}] = \min$$

DATA ASSIMILATION application:

**CSU-RAMS non-hydrostatic model: Total humidity mixing ratio (level=200m,
Nens=50, *Nstate*=54000)**

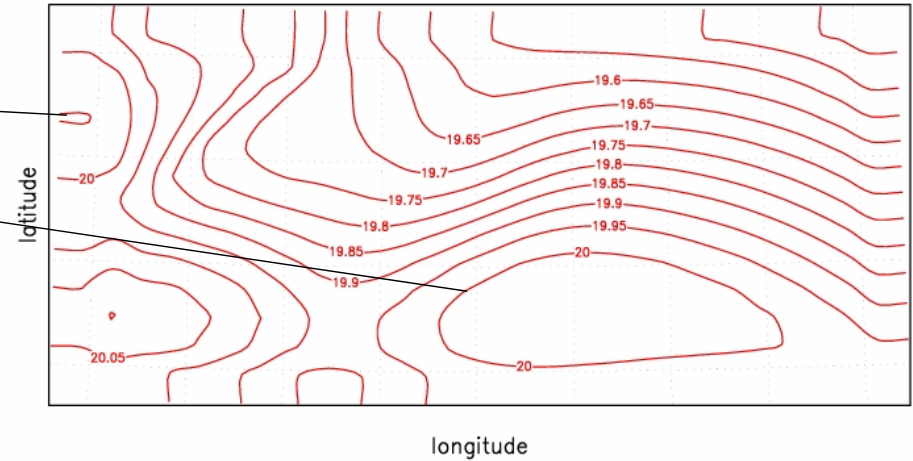
TRUTH

TRUE total humidity mixing ratio (g/kg)
cycle 31, level 200m



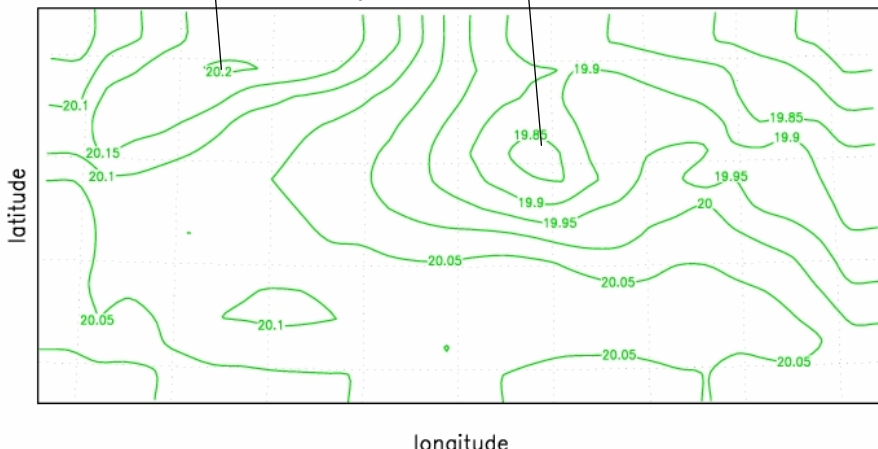
NO ASSIMILATION

NO ASSIMILATION total humidity mixing ratio (g/kg)
cycle 31, level 200m



ASSIMILATION

WITH ASSIMILATION total humidity mixing ratio (g/kg)
cycle 31, level 200m



**Locations of min and max centers
are much improved in the
experiment with assimilation.**