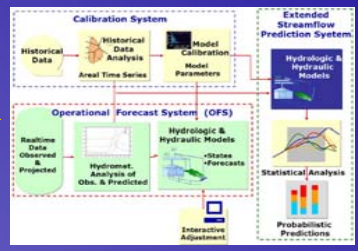


Another View to Hydrologic Ensemble Prediction via Sequential Monte Carlo Methods

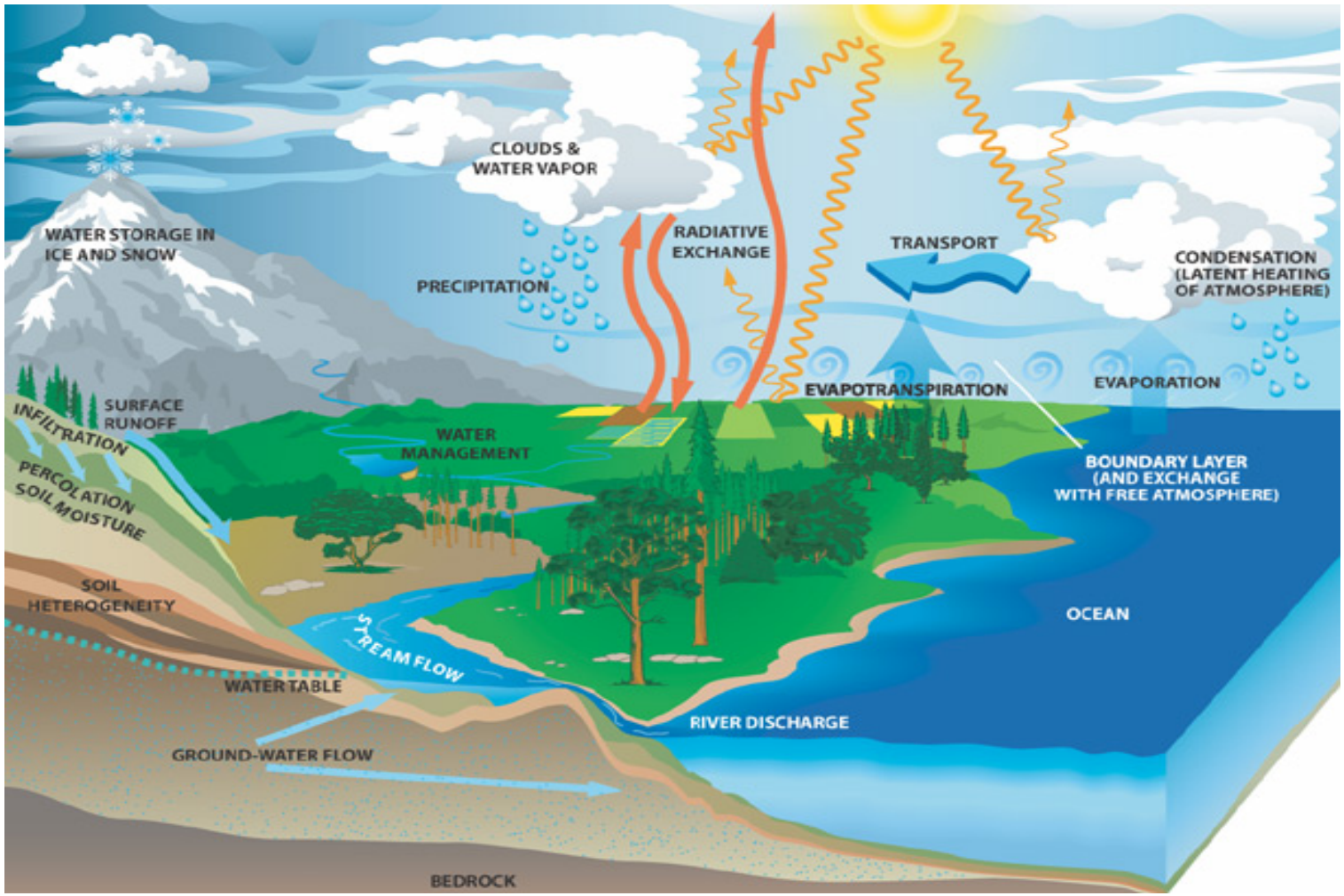
Hamid Moradkhani, K. Hsu and S. Sorooshian

Center for Hydrometeorology and Remote Sensing
University of California, Irvine

Outline

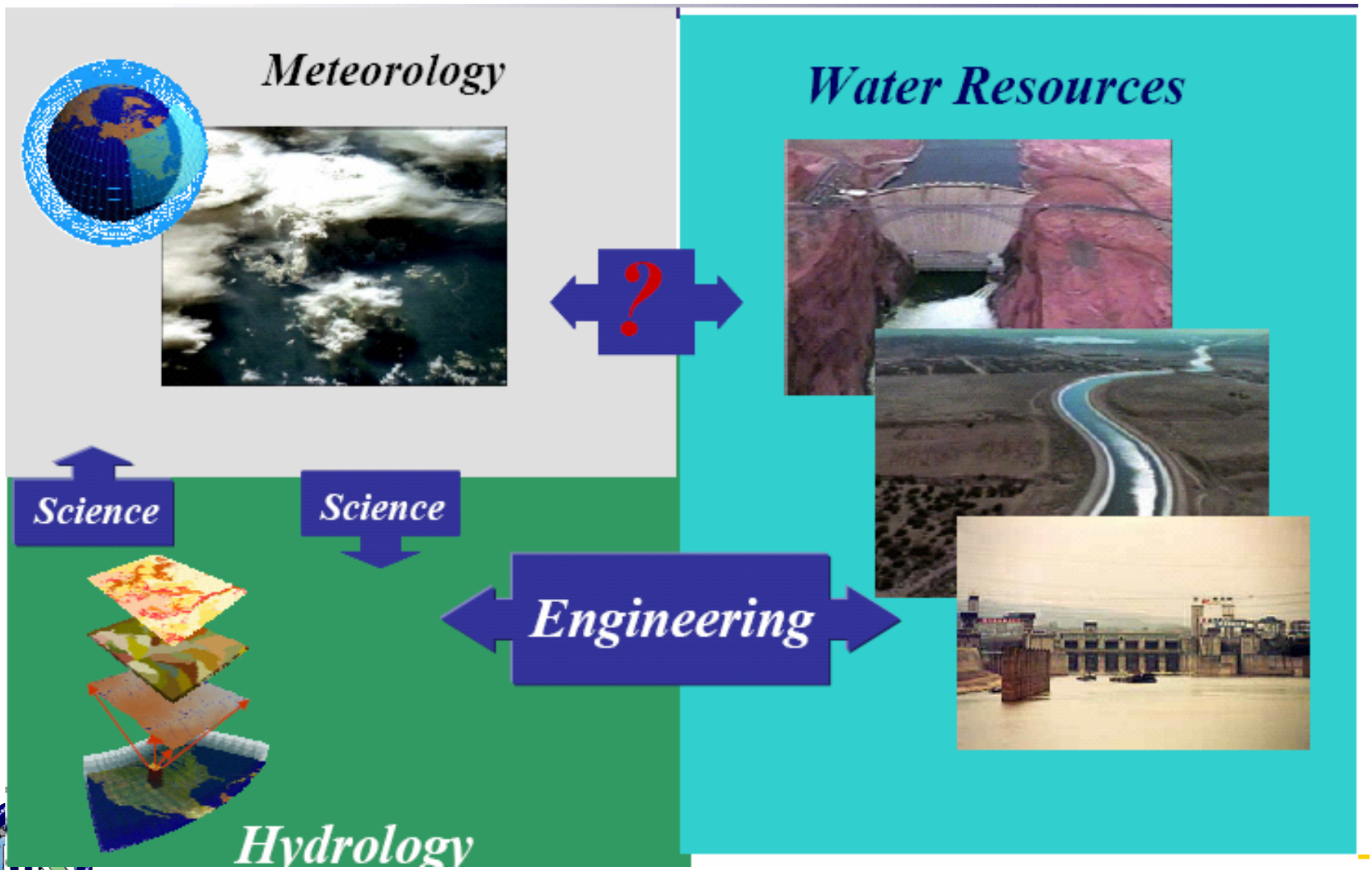
- Operational Forecasting System (AHPS)
- Hydrologic Model Calibration and Data Assimilation
- State-Parameter Uncertainty Estimation using Sequential Monte Carlo:
  - Dual Ensemble Kalman Filtering (DEnKF)
  - Particle Filtering (PF)

Hydrologic Cycle

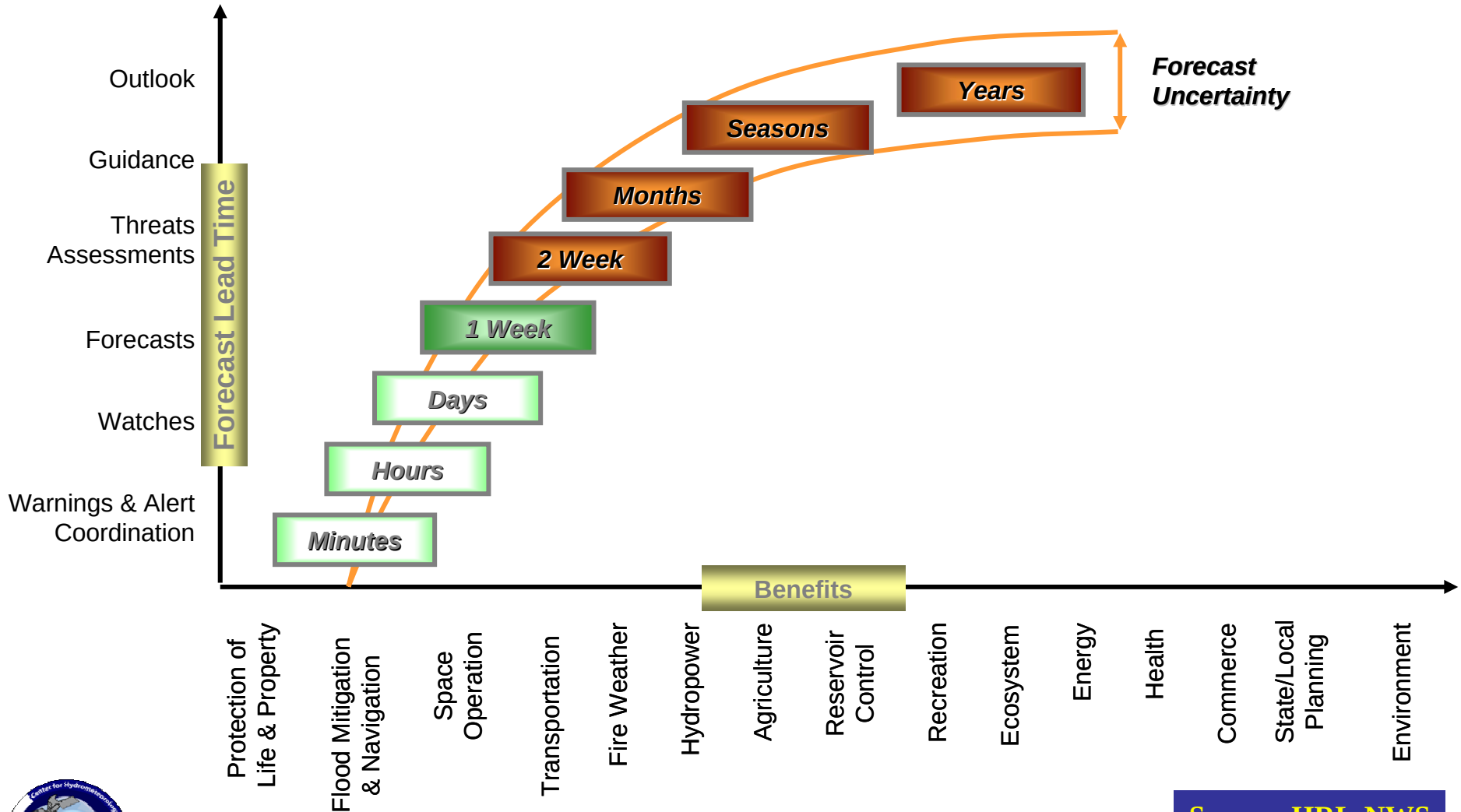


http://watercycle.gsfc.nasa.gov

Meteorology, Hydrology and Water Resources

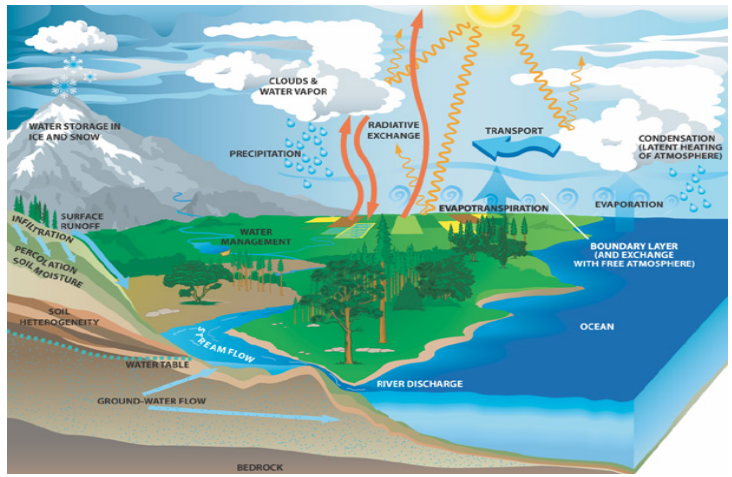


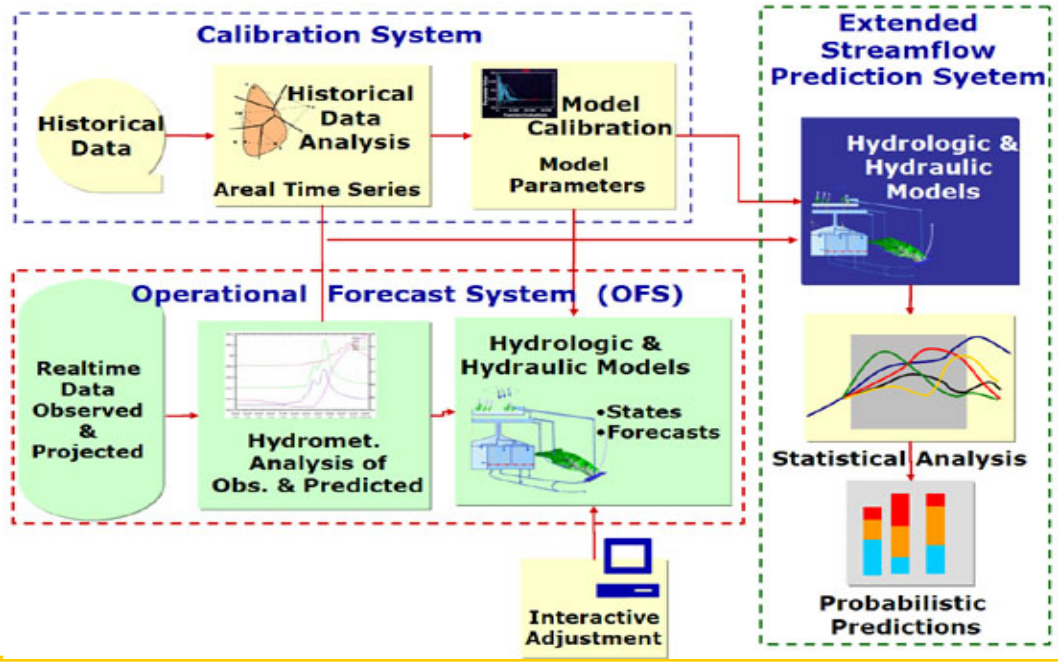
Seamless Suite of Forecasts



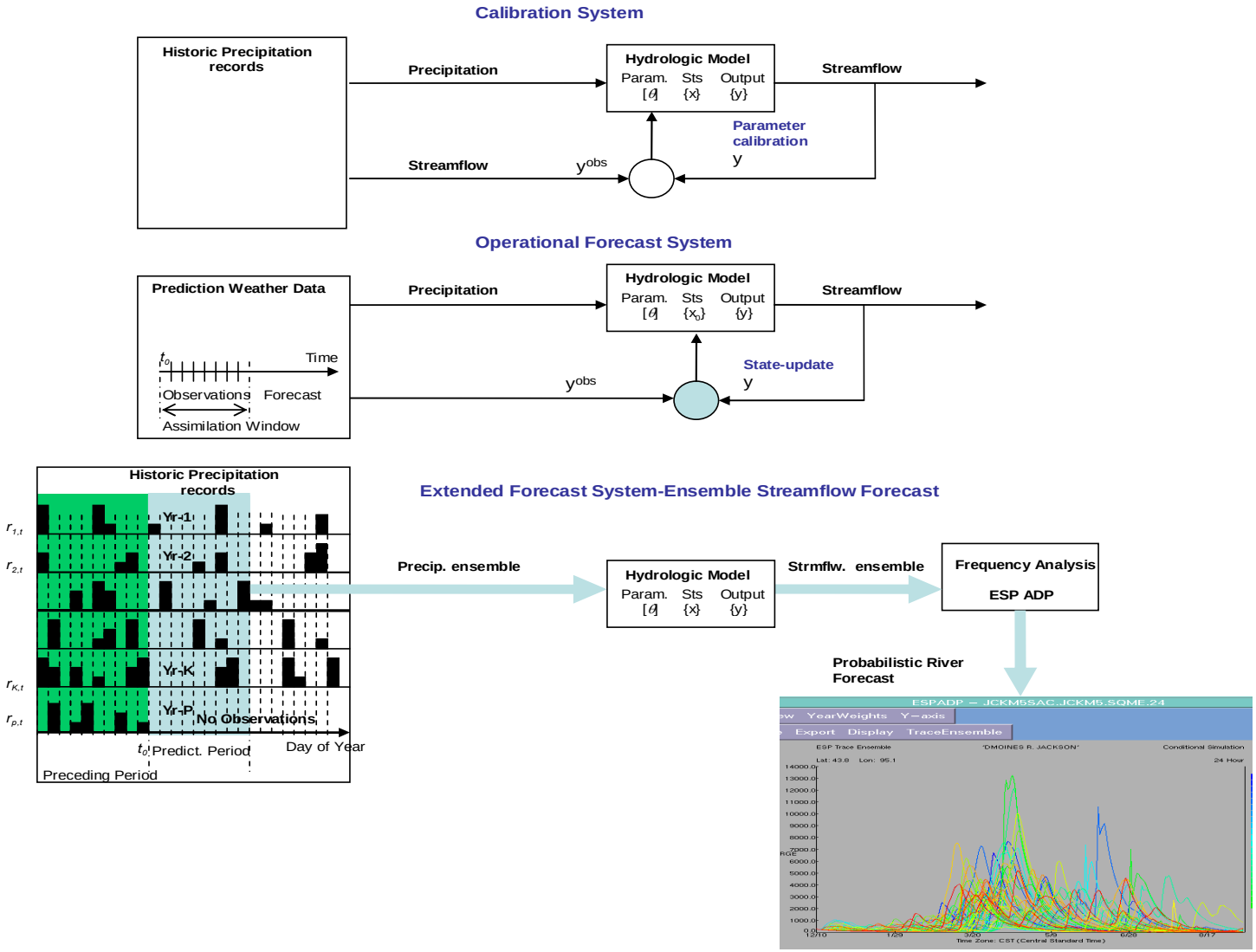
Source: HRL-NWS

Advanced Hydrologic Prediction System (AHPS)

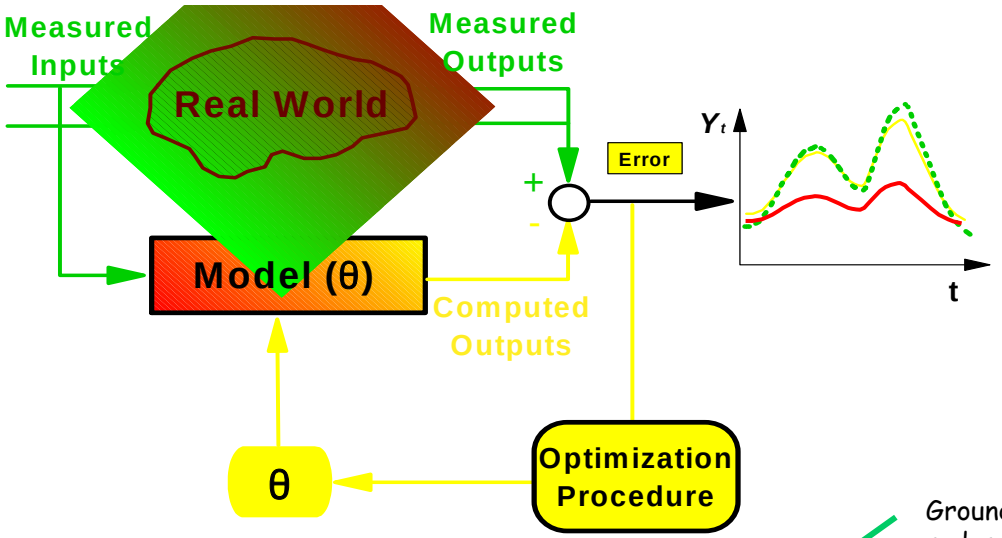


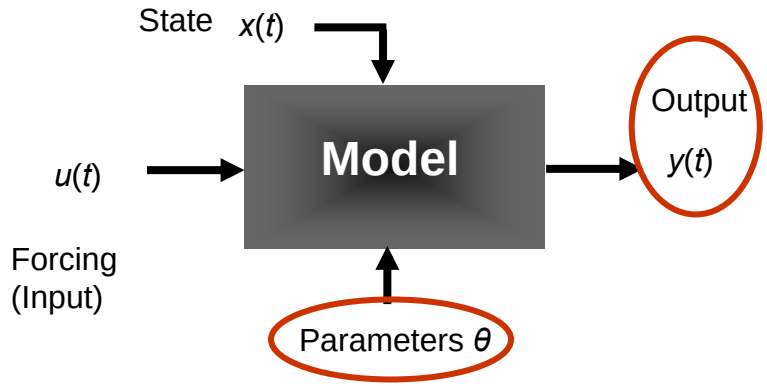


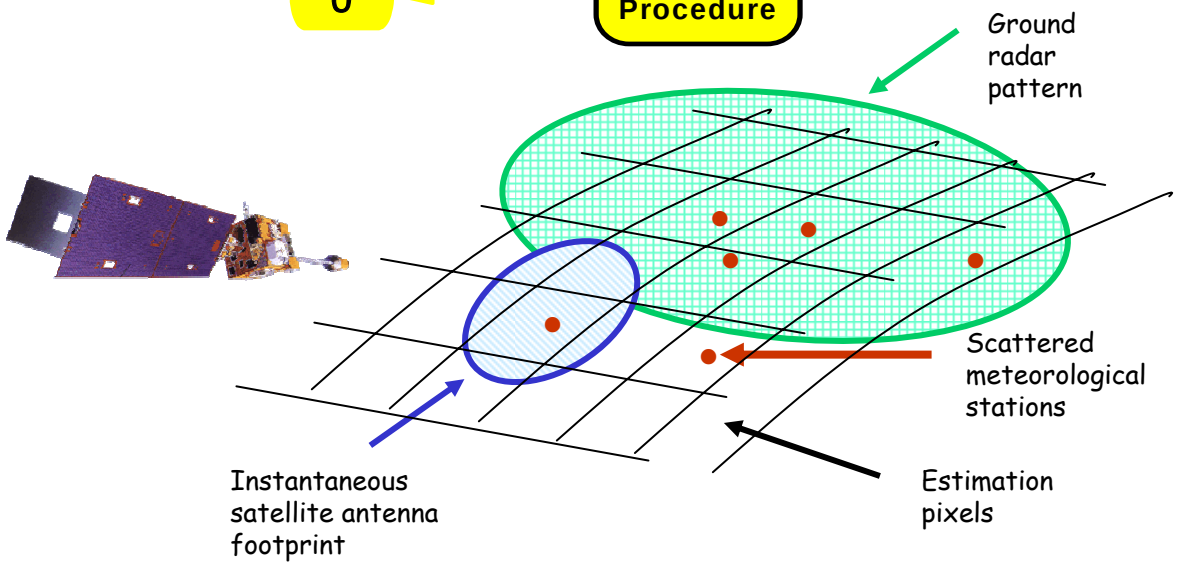
Ensemble Streamflow Prediction in AHPS



Model Calibration / Data Assimilation





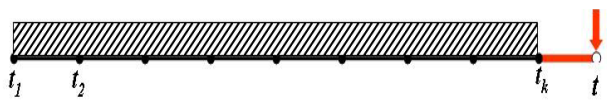


McLaughlin, 2001

Filtering (Nowcasting)/Forecasting:

Characterize system over time interval $t \geq t_k$.

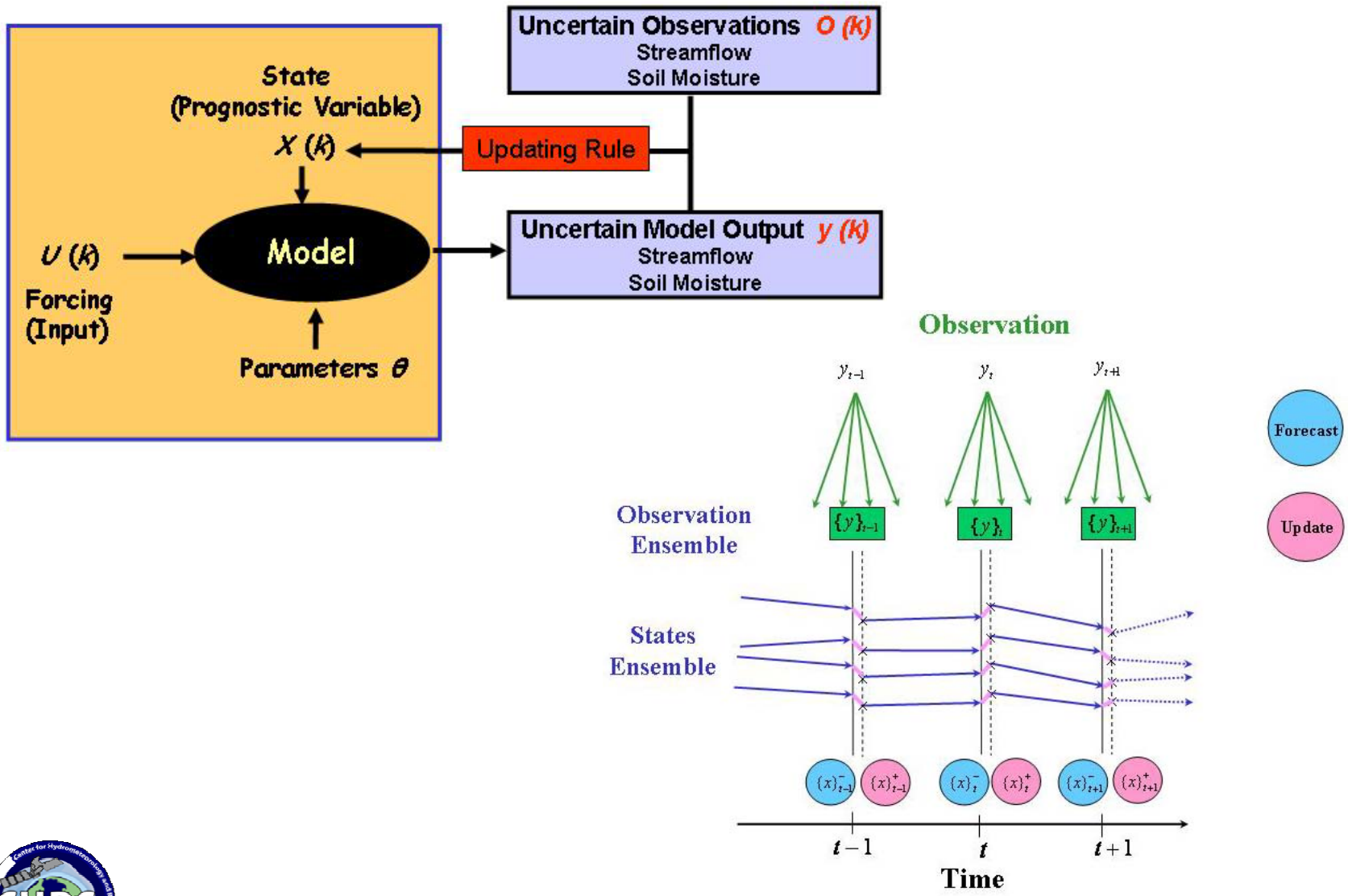
Used for real-time forecasting/updating



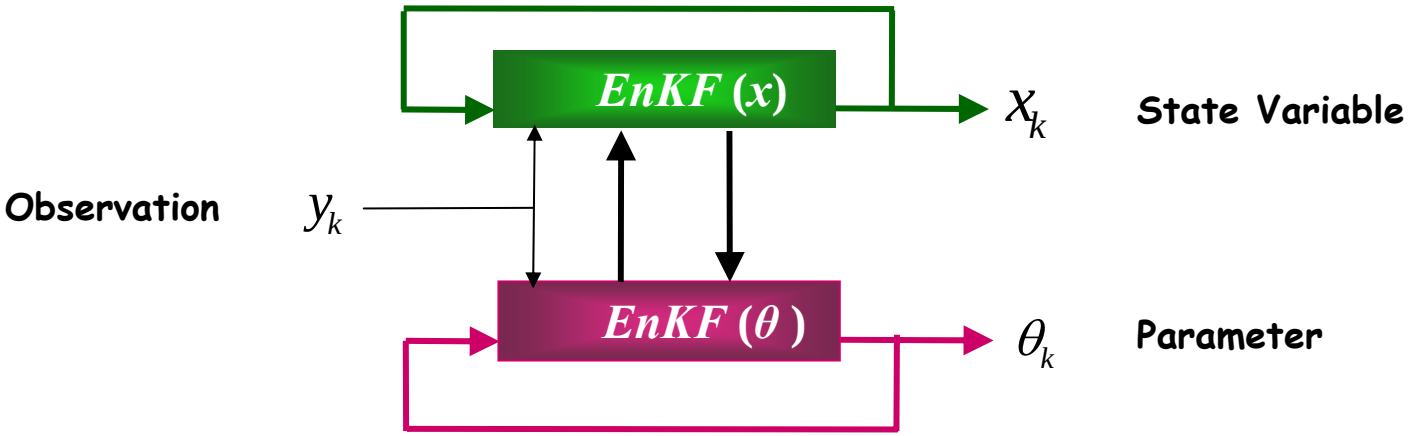
State-Parameter Uncertainty Estimation using Sequential Monte Carlo:

Dual Ensemble Kalman Filtering (EnKF)

Ensemble Kalman Filter



Dual State-Parameter Estimation via EnKF



Parameter Forecast (Random Walk)

$$\theta_{k+1}^{i-} = \theta_k^{i+} + \gamma_k^i \qquad \gamma \sim N(0, \Sigma^\theta)$$

Parameter Updating

$$\theta_{k+1}^{i+} = \theta_{k+1}^{i-} + K_{k+1}^{\theta}(y_{k+1}^i - \hat{y}_{k+1}^i)$$

State Forecast

$$x_{k+1}^{i-} = f(x_k^{i+}, \theta_{k+1}^{i+}, u_k^i)$$

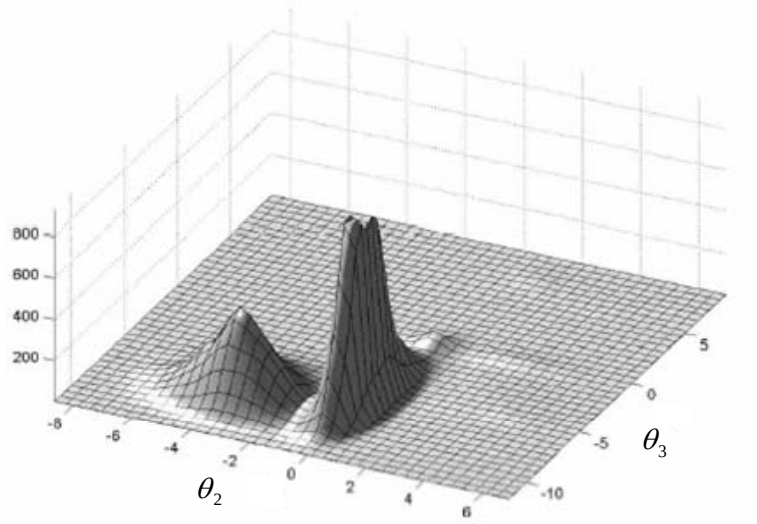
State Updating

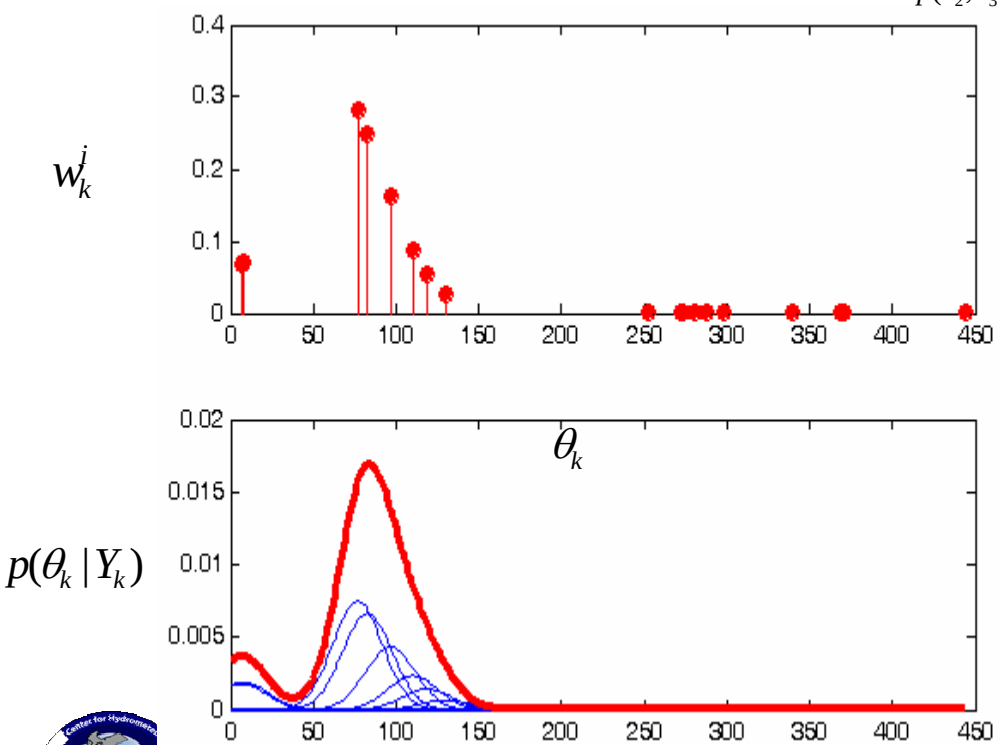
$$x_{t+1}^{i+} = x_{k+1}^{i-} + K_{k+1}^{x}(y_{k+1}^i - \hat{y}_{k+1}^i)$$

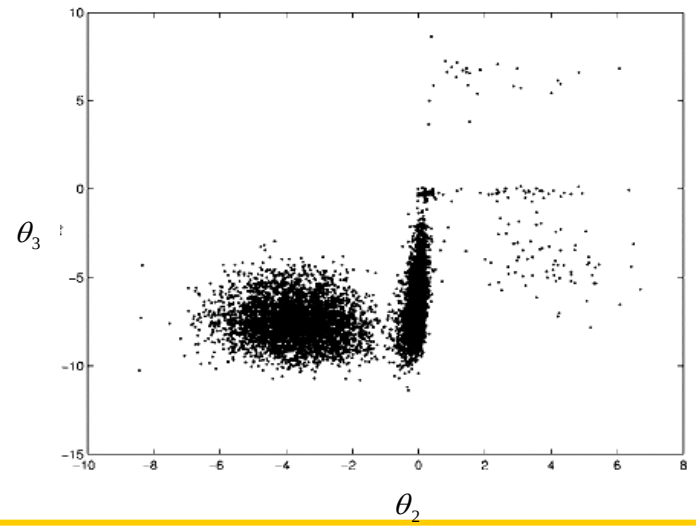
Kernel Smoothing used in the EnKF

$$P(\theta_{k+1} \mid y_1, \ldots, y_k) \sim \sum_{i=1}^{n} w_k^i N(\theta_{k+1} \mid m_k^i, h^2 V_k)$$

$$\theta_{k+1}^{i-} \sim N(. \mid a\theta_k^{i+} + (1-a)\bar{\theta}^+, h^2 \sum_k^{\theta+})$$

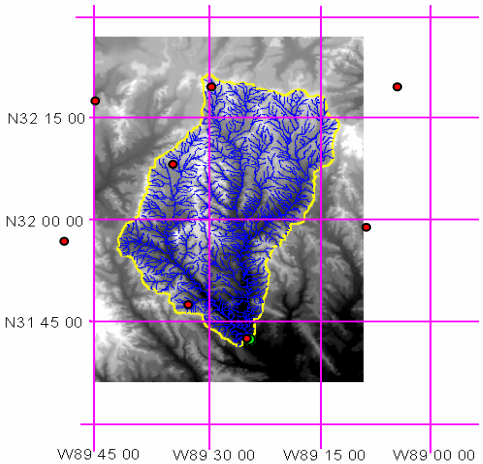


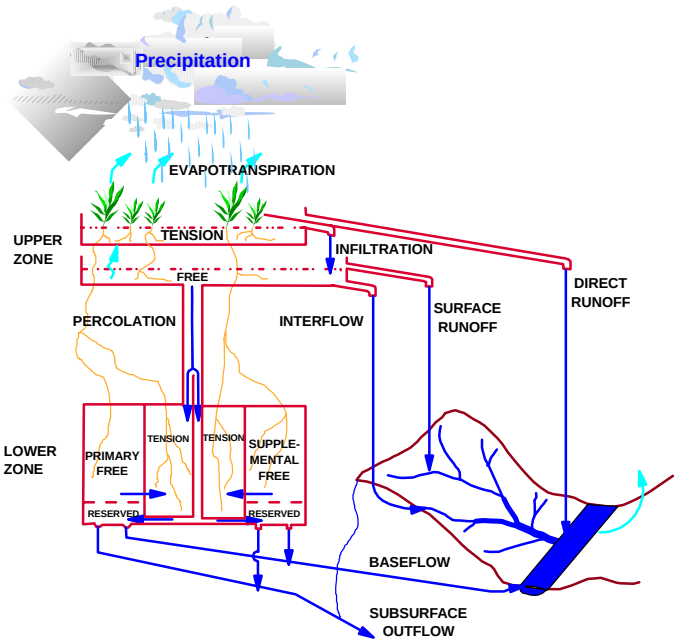


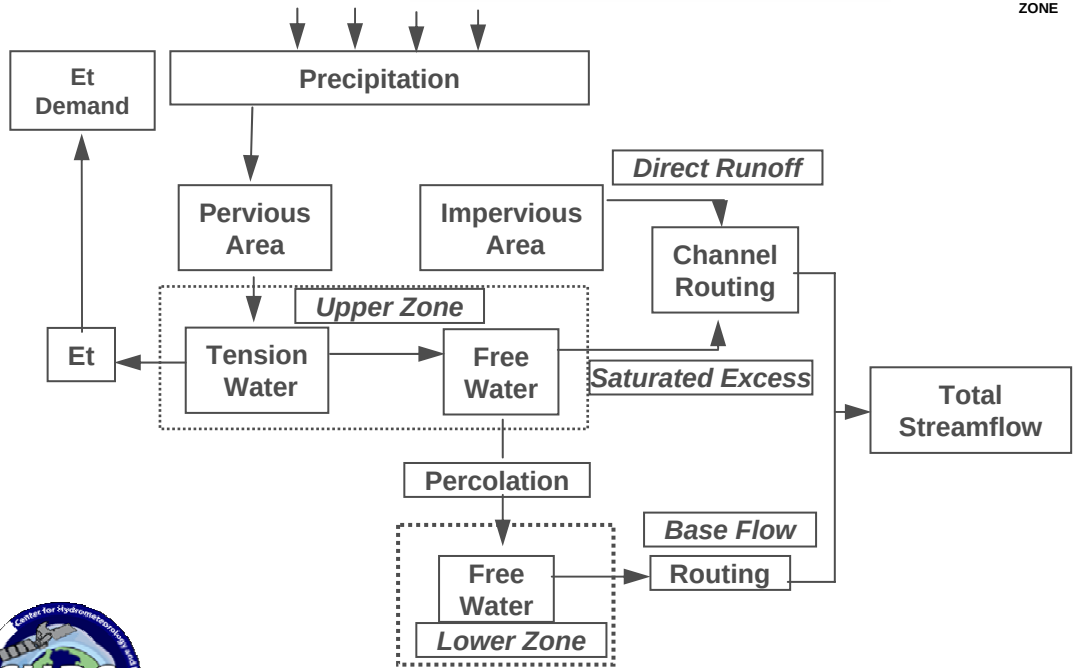


Study Area and Hydrologic Model

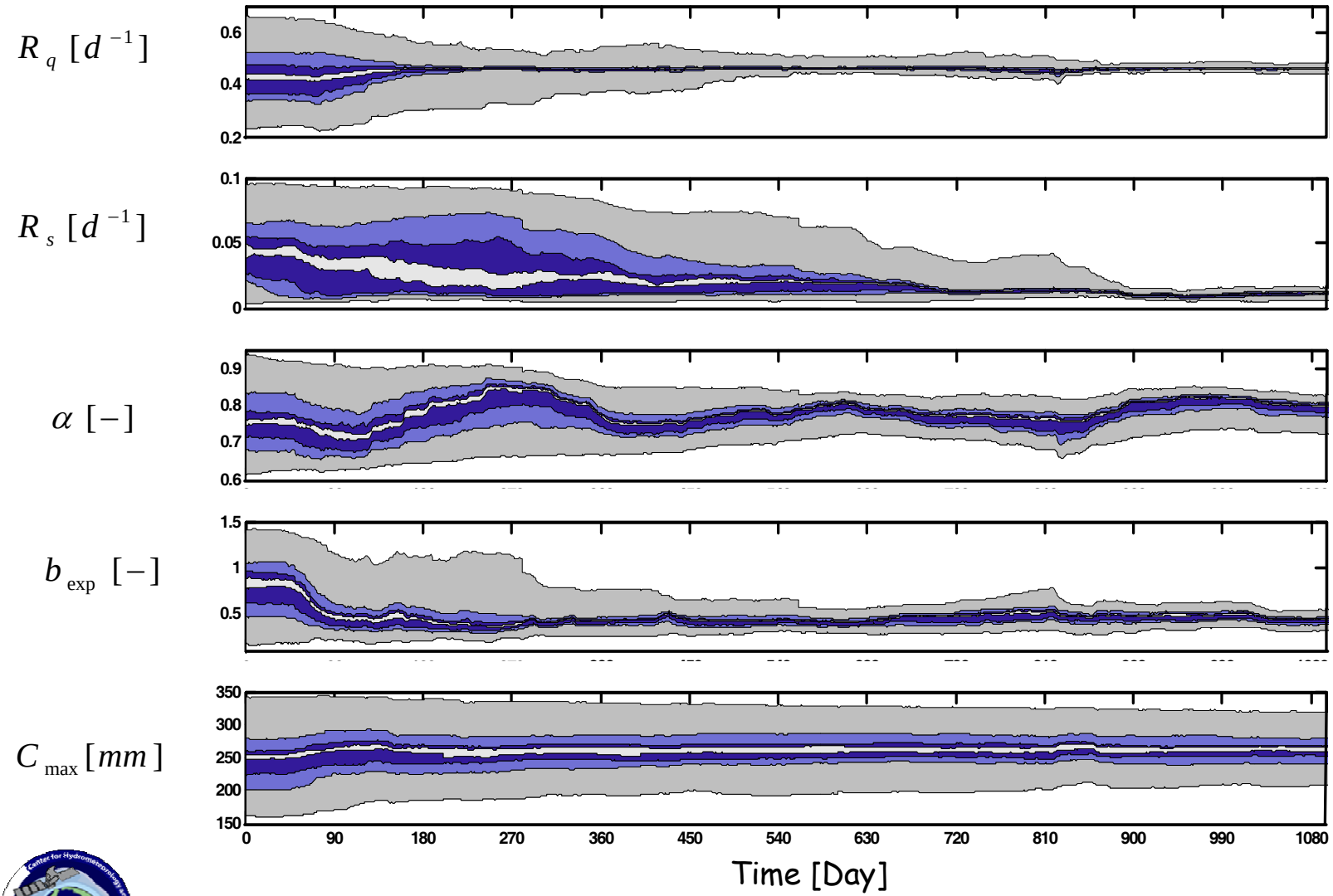
Leaf River Basin Near North of Collins, Mississippi Basin Area : 1949 km^2



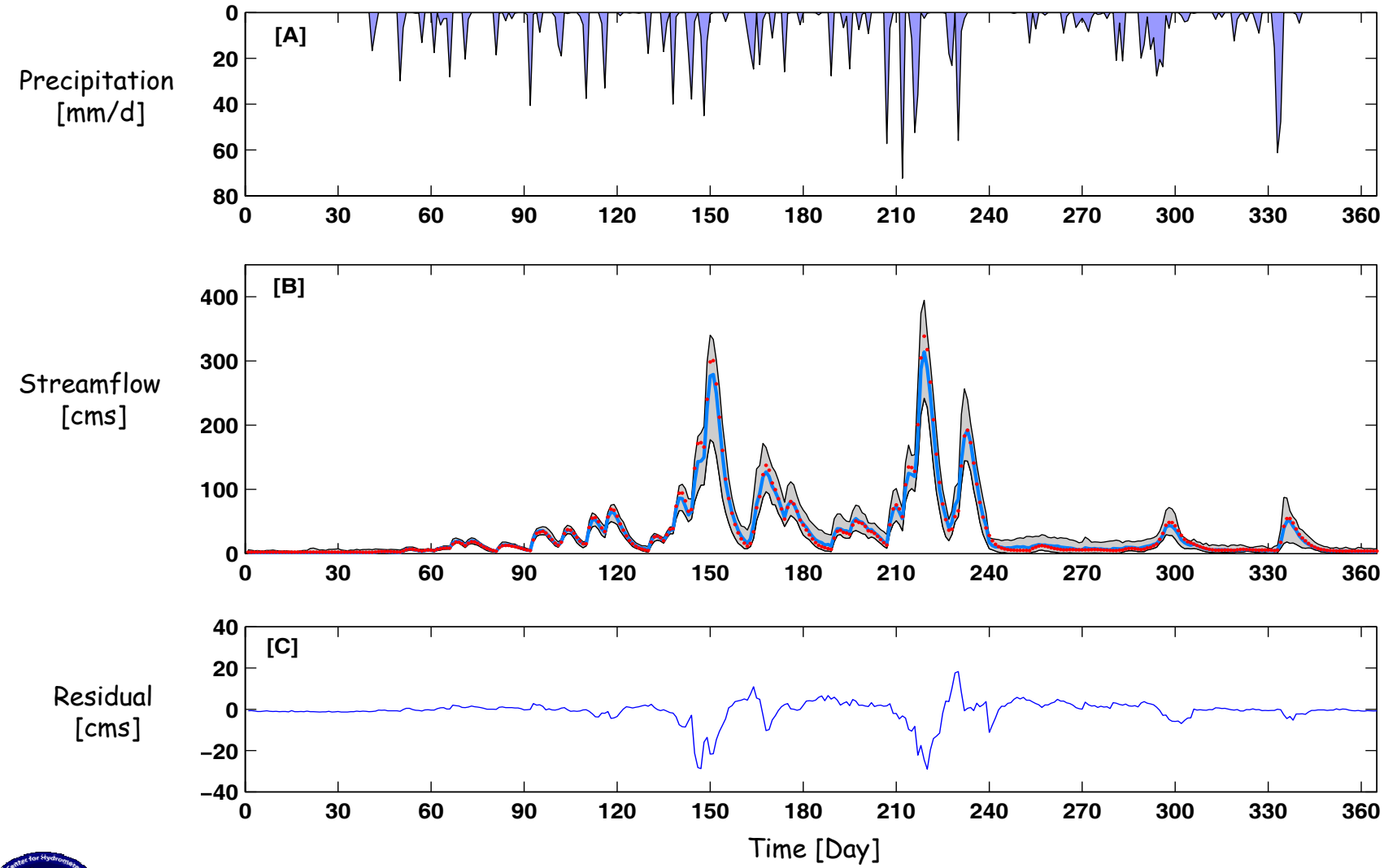




Uncertainty Evolution of HyMOD Parameters via Dual EnKF



Uncertainty in Streamflow Forecasting (Dual EnKF)

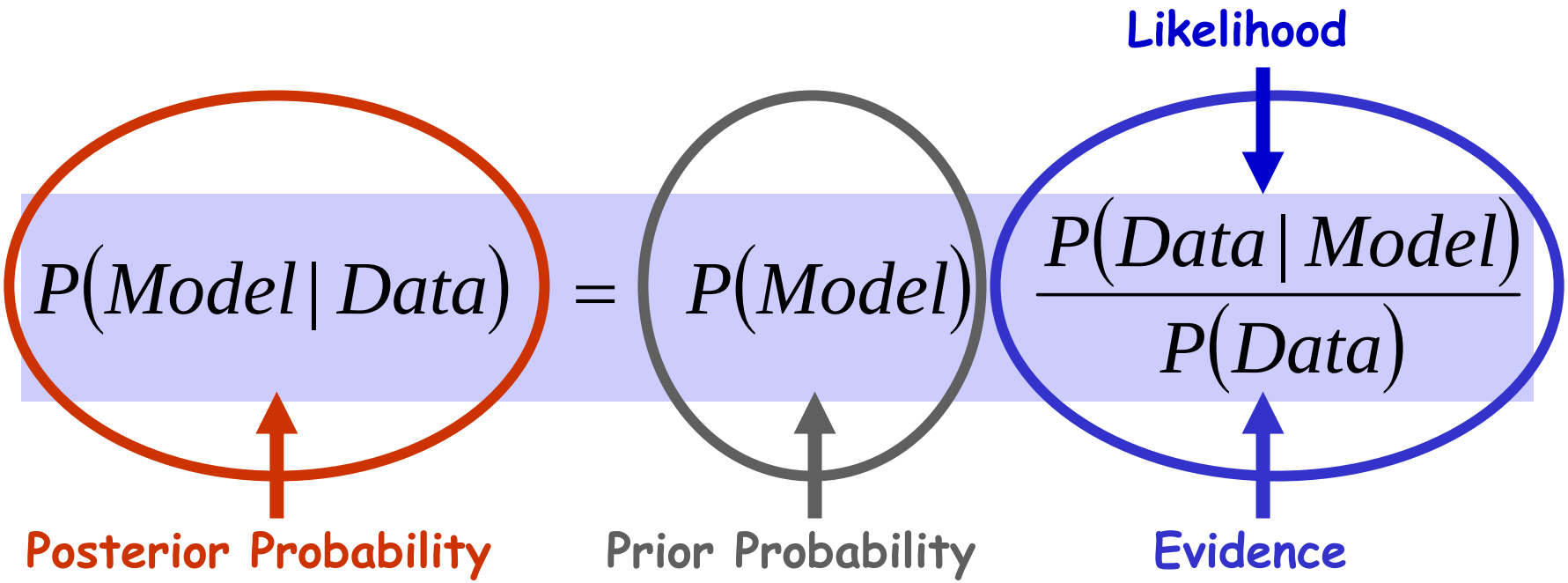


State-Parameter Uncertainty Estimation using Sequential Monte Carlo:

Bayesian Recursive Estimation via Particle Filtering (PF)

Implementation of Bayes Law

The **Prior Probability describes what you first knew**. Multiply this by a **term that describes the effect of your new information**, and the result is what you know **after you have taken into account your new data.**



Bayesian Recursive Estimation

Likelihood

Forecast Density (Prior)

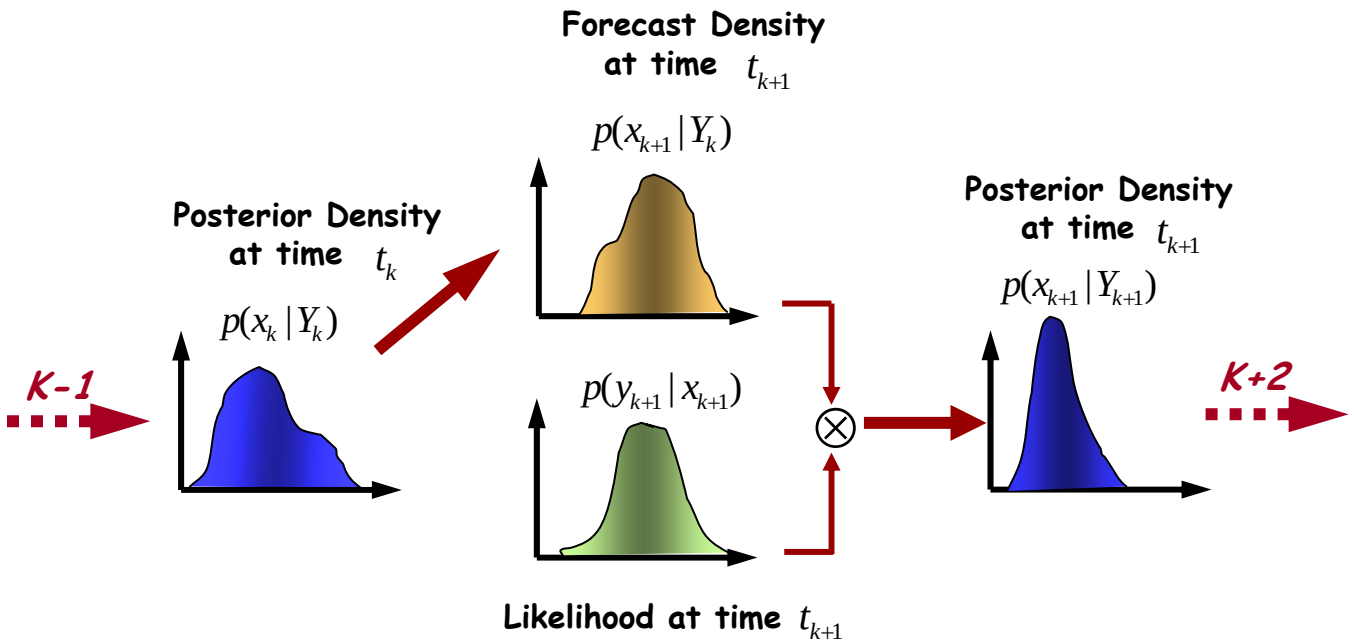
Analysis Density (Posterior)

$$p(x_{k+1} \mid Y_{k+1}) = \frac{p(y_{k+1} \mid x_{k+1})\, p(x_{k+1} \mid Y_k)}{p(y_{k+1} \mid Y_k)}$$

Evidence

$$p(x_{k+1} \mid Y_k) = \int_{x_k} p(x_{k+1} \mid x_k)\, p(x_k \mid Y_k)\, dx_k$$

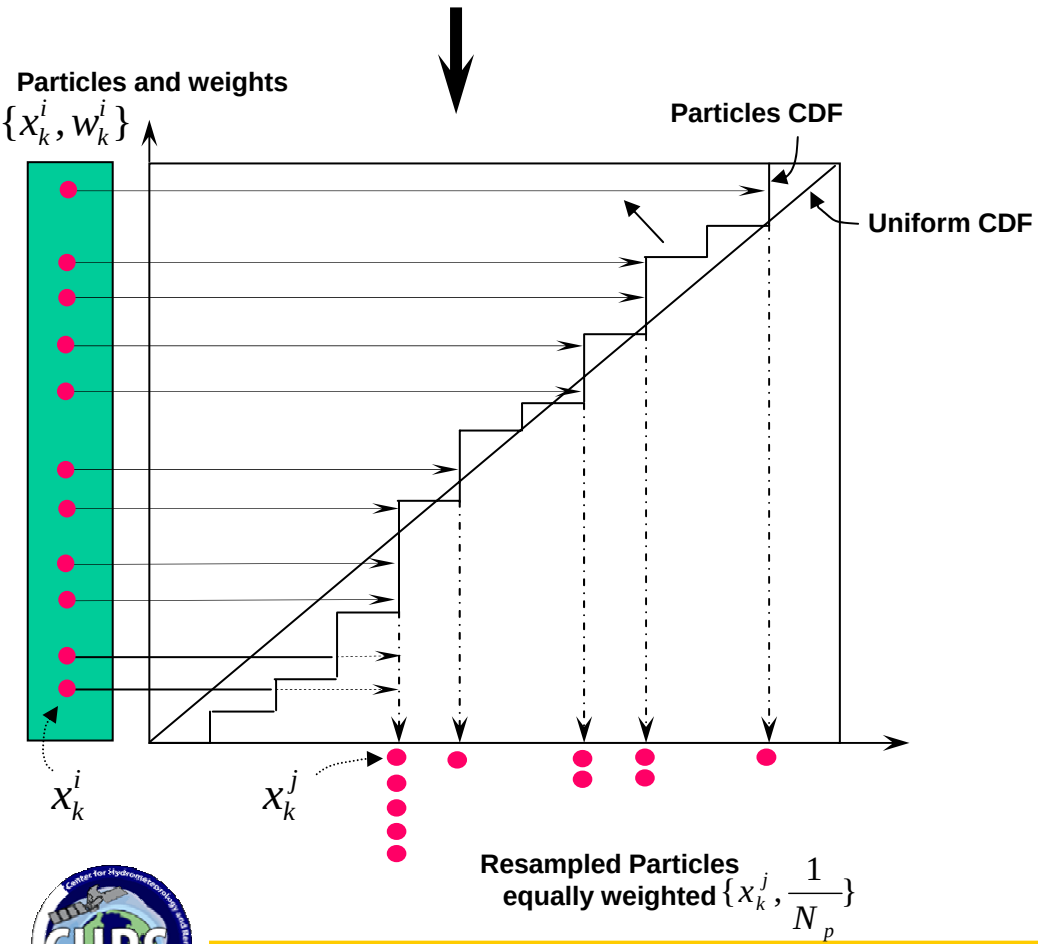
$$p(y_{k+1} \mid Y_k) = \int_{x_{k+1}} p(y_{k+1} \mid x_{k+1})\, p(x_{k+1} \mid Y_k)\, dx_{k+1}$$

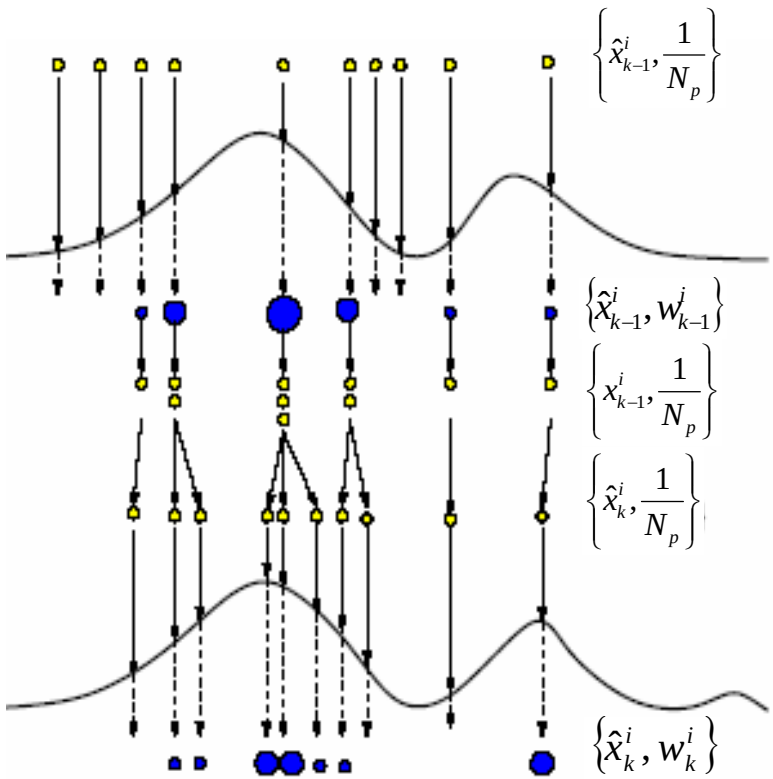


Particle Filtering in State-Space

1) Posterior $p(x_{k+1} \mid Y_{k+1}) = \sum_{i=1}^{N_p} w_{k+1}^{i} \delta(x_{k+1} - x_{k+1}^{i})$ $\quad w_{k+1}^{i*} \propto w_k^{i*} \dfrac{p(y_{k+1} \mid x_{k+1}^{i}) p(x_{k+1}^{i} \mid x_k^{i})}{q(x_{k+1}^{i} \mid x_k^{i}, y_{k+1})}$

2) Sequential Importance Resampling (SIR) $\quad q(x_{k+1}^{i} \mid x_k^{i}, y_{k+1}) = p(x_{k+1}^{i} \mid x_k^{i})$





Van der Merwe et al. 2000

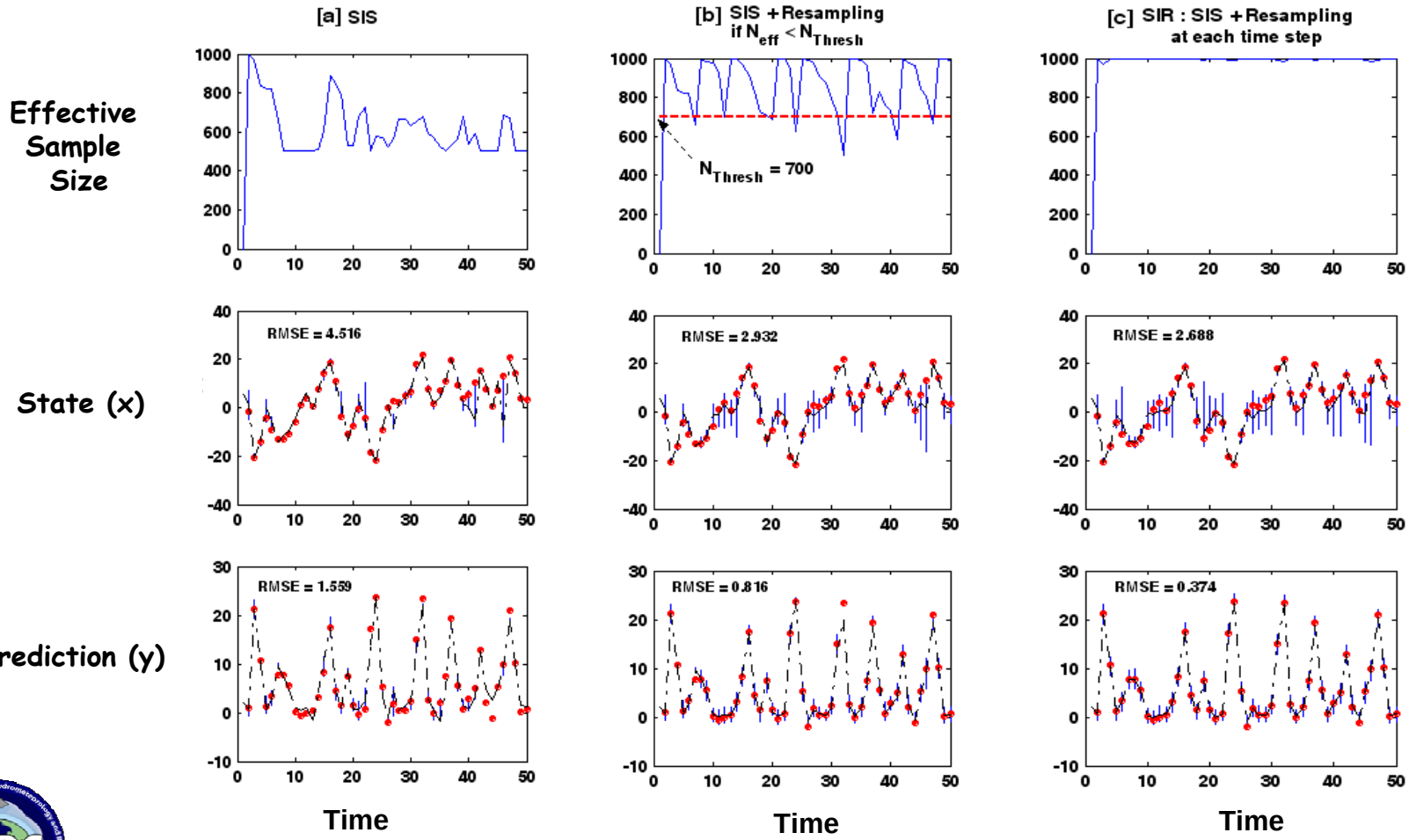
Example (SIS vs. SIR)

State Equation

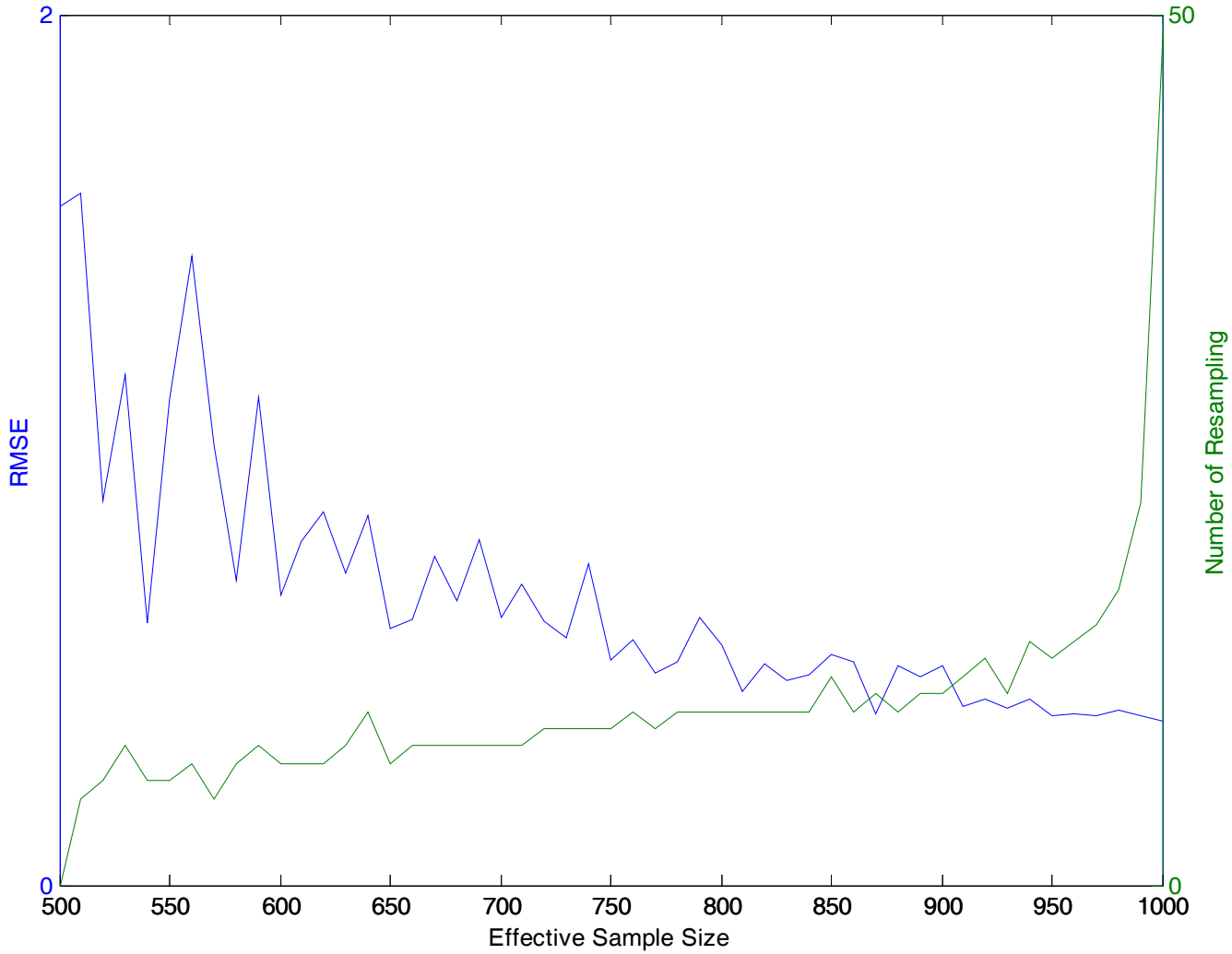
$$x_k = \frac{1}{2}x_{k-1} + 25\frac{x_{k-1}}{1+x_{k-1}^2} + 8\cos(1.2k) + \omega_k$$

Observation Equation

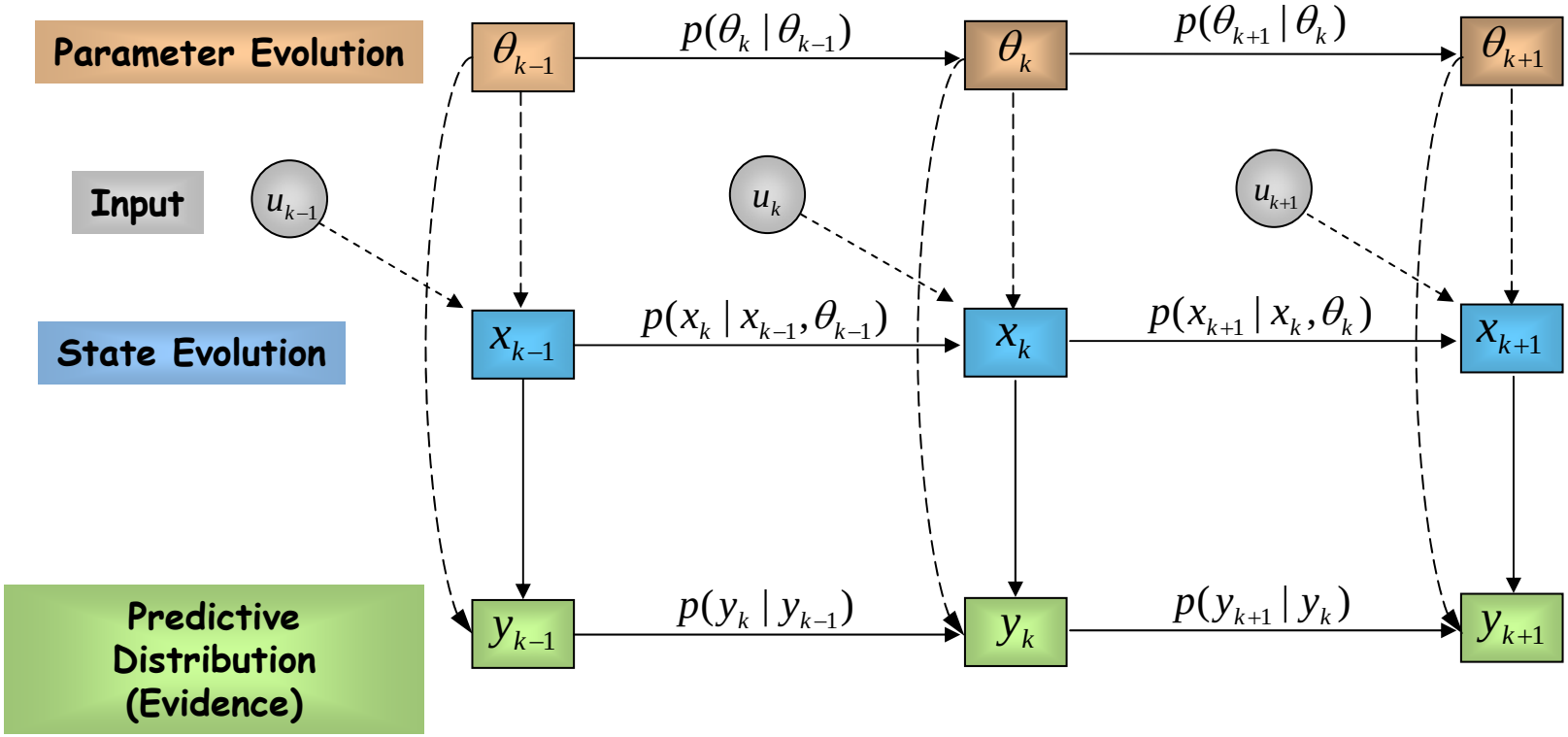
$$y_k = \frac{x_k^2}{20} + \nu_k$$



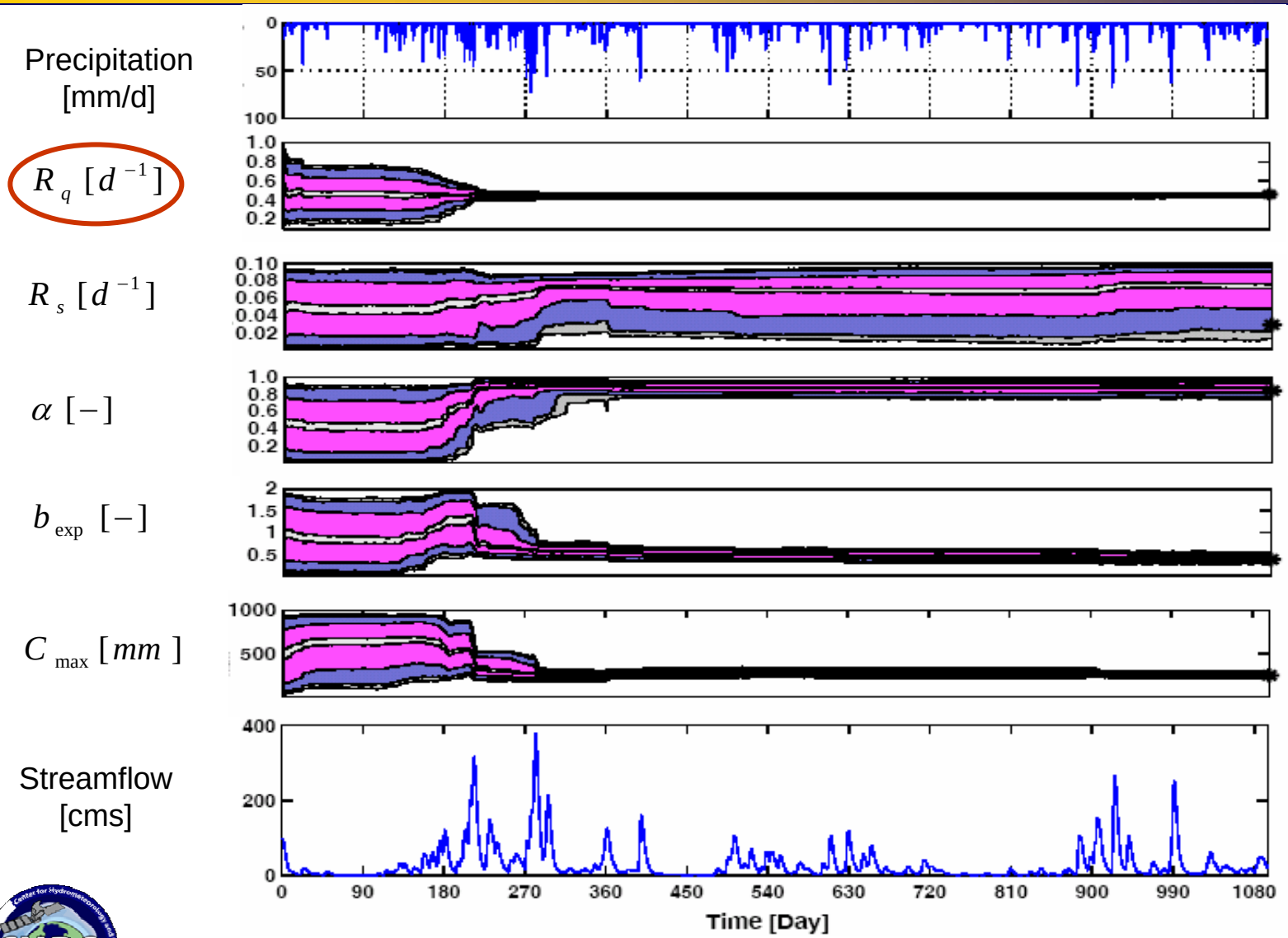
Performance vs. Efficiency (filtering for 50 time steps)



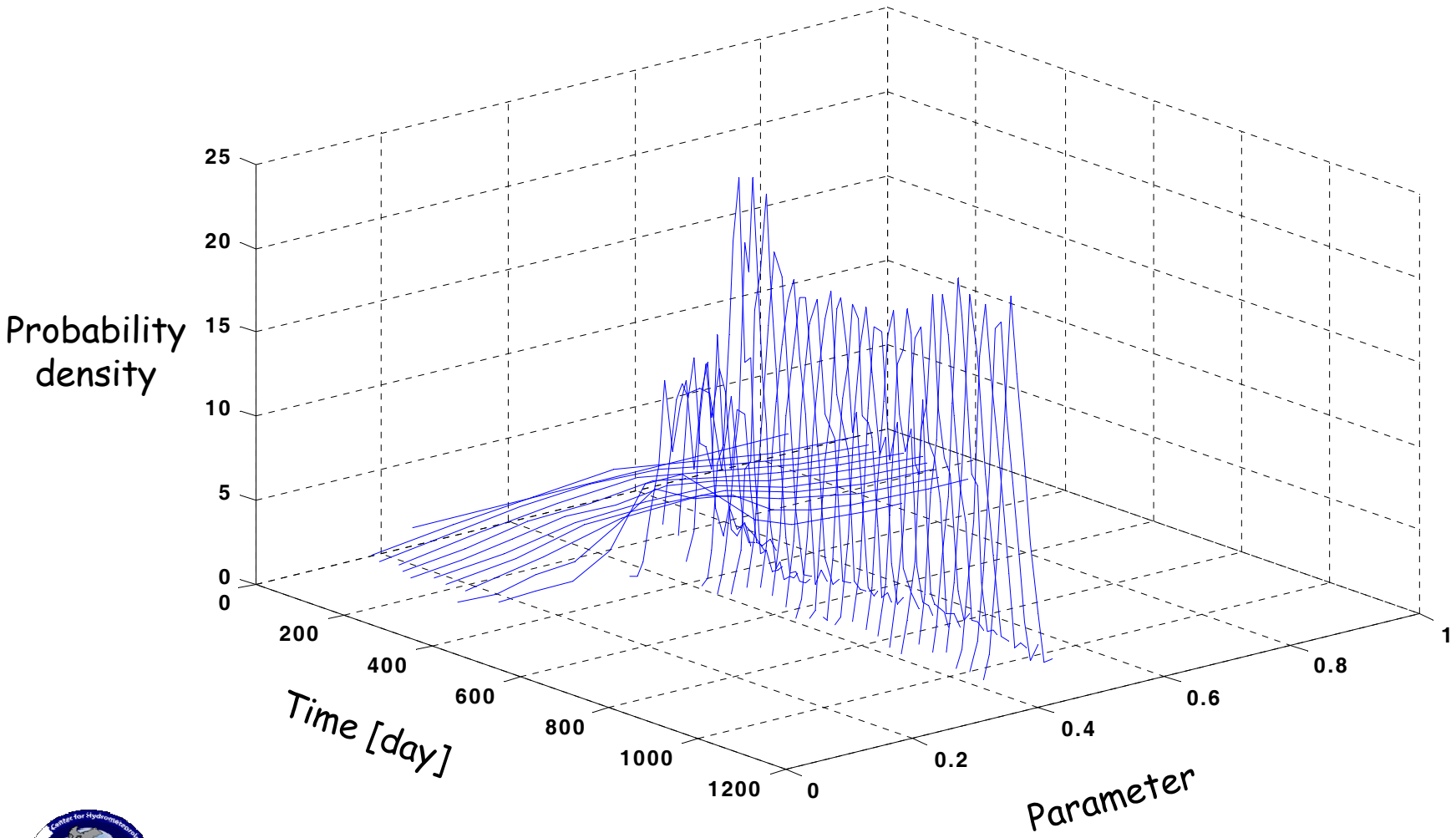
Bayesian Recursive State-Parameter Estimation



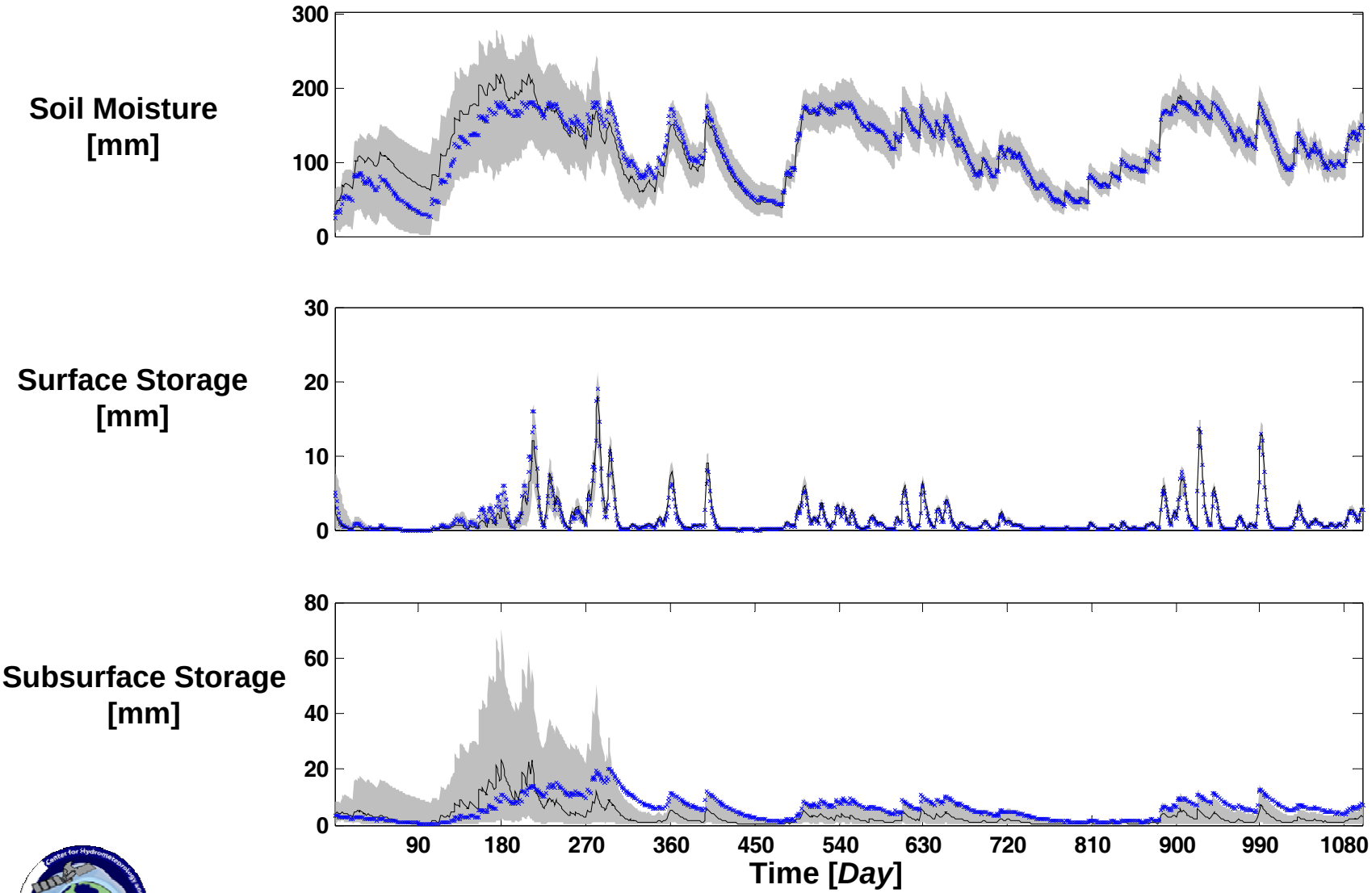
Uncertainty Evolution of HyMOD Parameters via PF



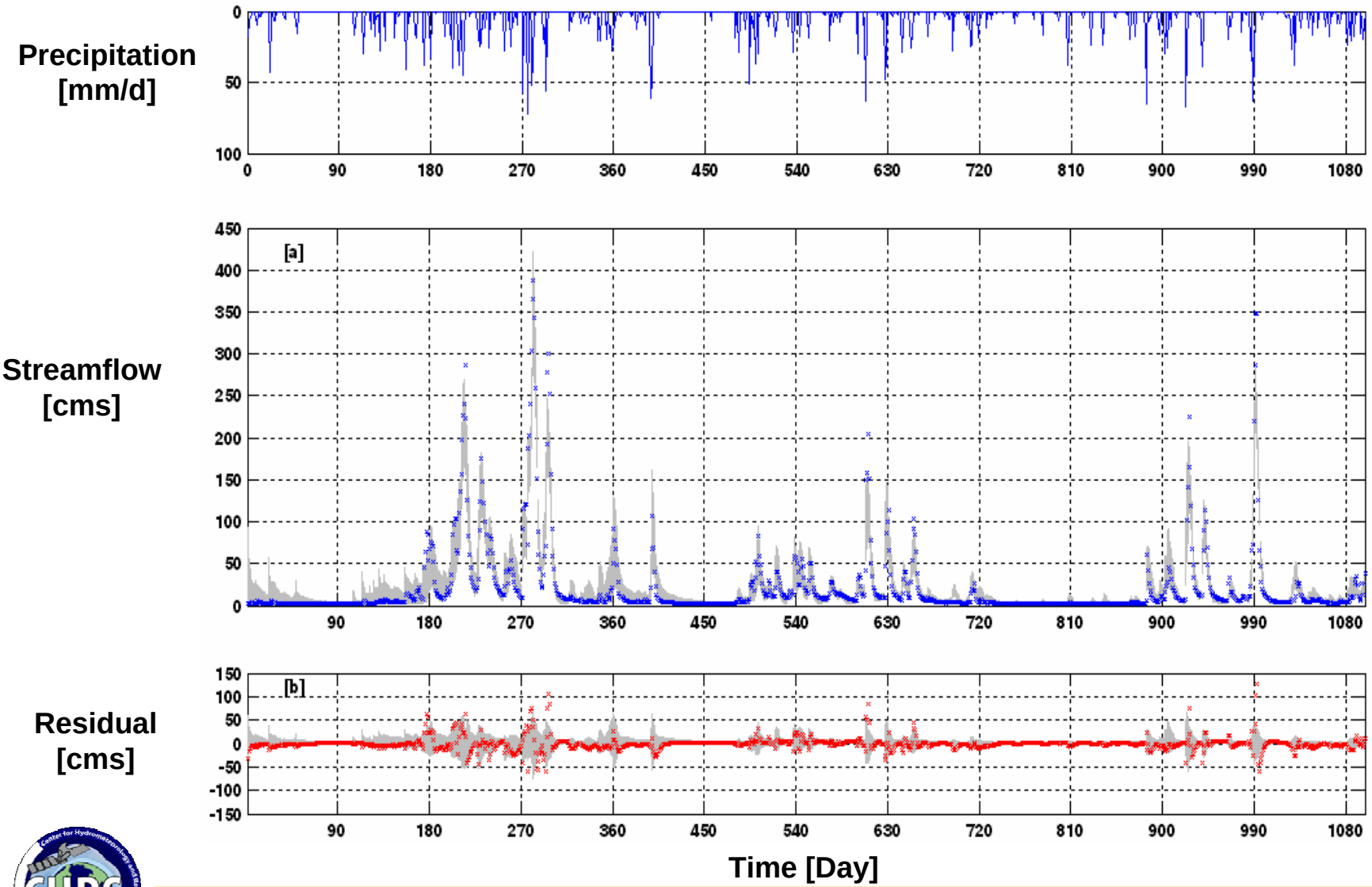
Evolution of a Parameter Probability Density



Uncertainty Bound Tracking for State Variables



Hydrograph Prediction with Uncertainty Estimation



Summary and Conclusion

- *Ensemble Data Assimilation can deal with all sources of uncertainties in the hydrologic forecasting*
- *Recursive state-parameter estimation could be embedded into the AHPS system which provides* not only *the parameter uncertainty but also the uncertain initial condition*
- *EnKF and PF have the capability of doing simultaneous state-parameter estimation*
- *PF has the superior feature over other assimilation techniques for adaptive inference of the Complete Posterior Distribution of variables on interest*