From Ensemble to Tree

An information based approach

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HEPEX International workshop on Post-processing and Verification of hydrological ensemble predictions

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Brief Personal Presentation

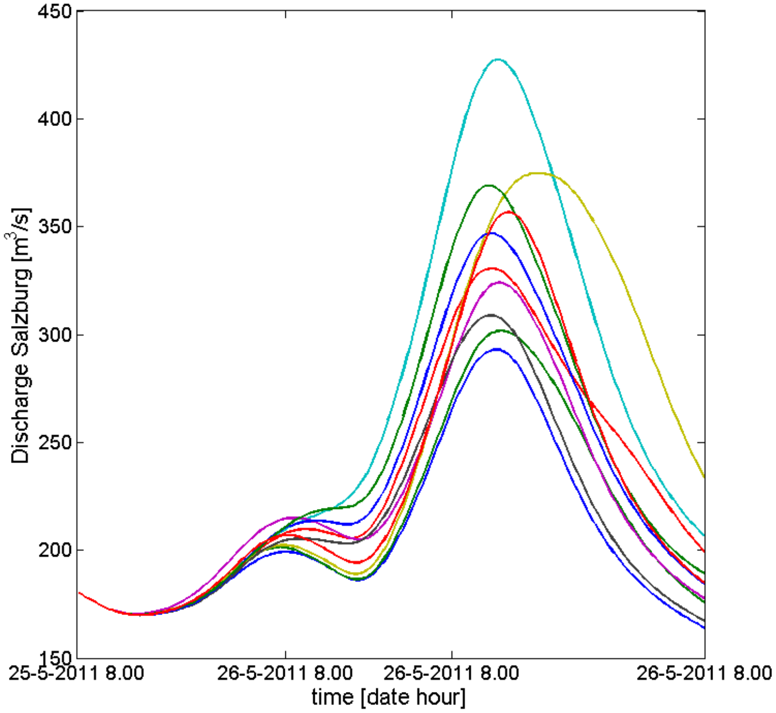
Current Research area:

- Optimal anticipatory control of water systems under uncertainty
  - Forecast uncertainty

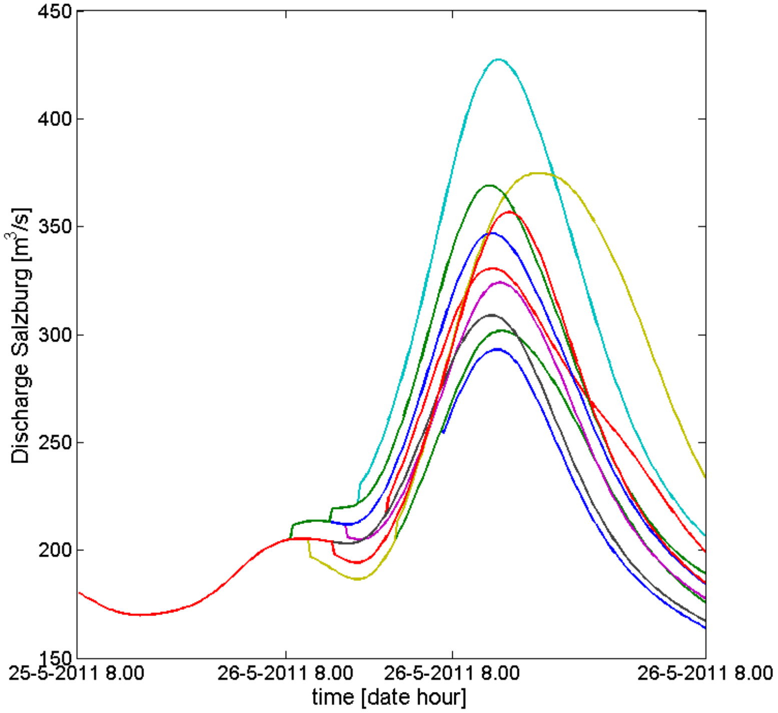
What is a tree?

Difference between a tree and an ensemble

- Tree and Ensemble are two different models of uncertainty



ensemble



tree

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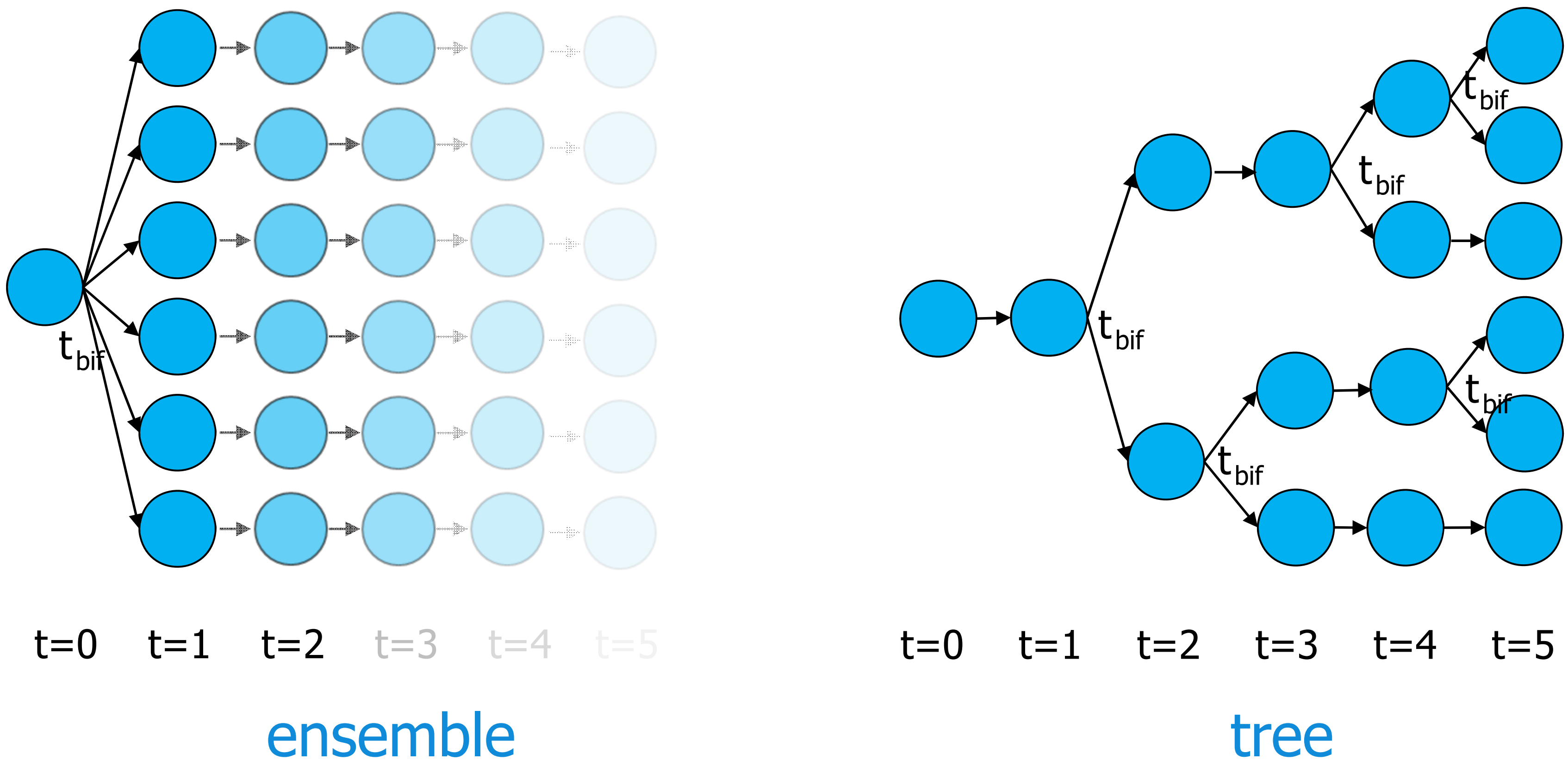

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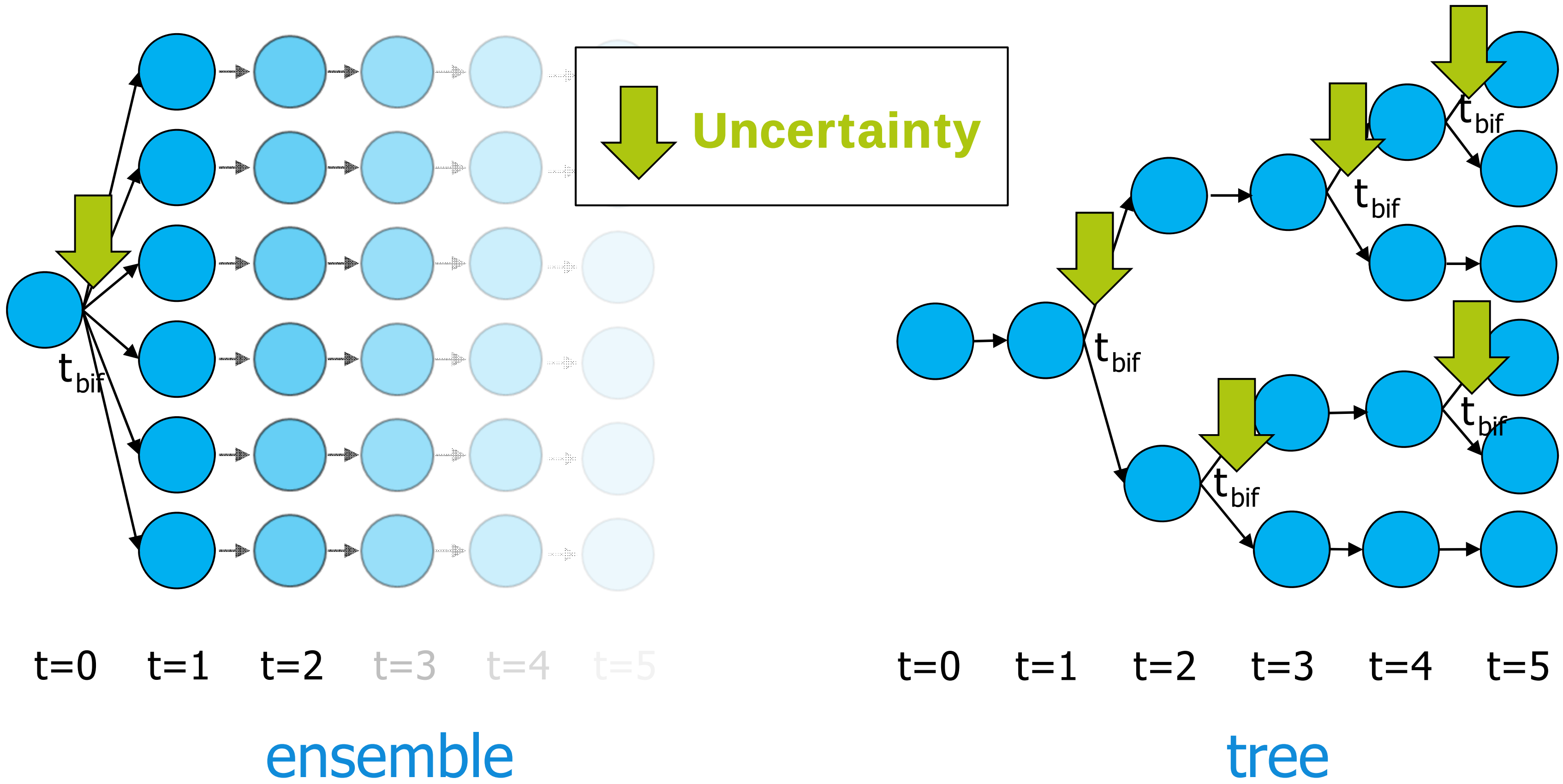


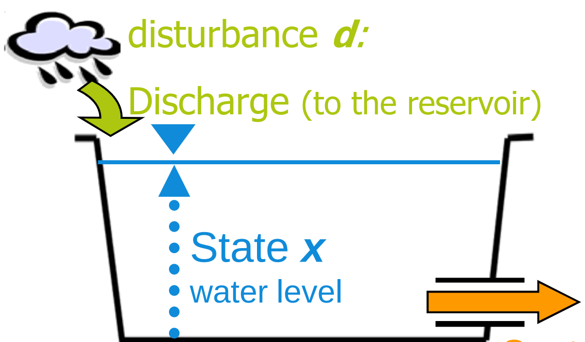
Why using a tree

The “two-stage” optimization problem

Two interdependent decisions, under uncertainty, in this sequence:

Decision | Uncertainty | Decision

Example: control of a reservoir



Release at t_1	Precipitation event	Release at t_2
u_1	D:[0,1]; P($d = 1$) $= p$	u_2

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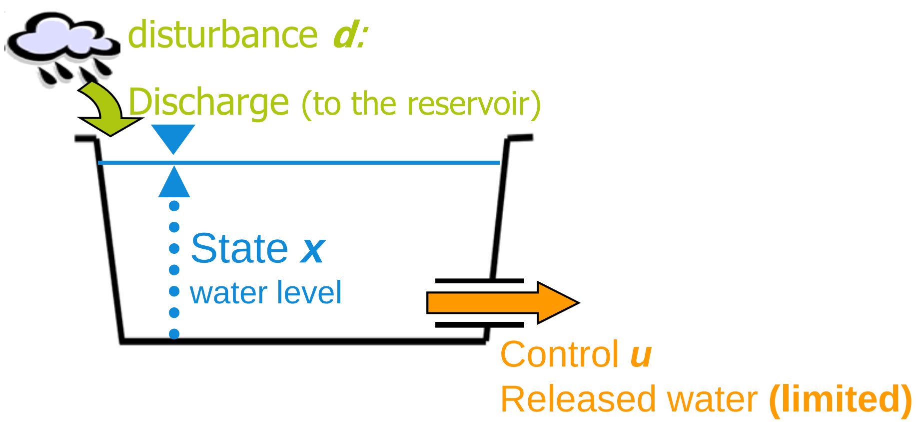

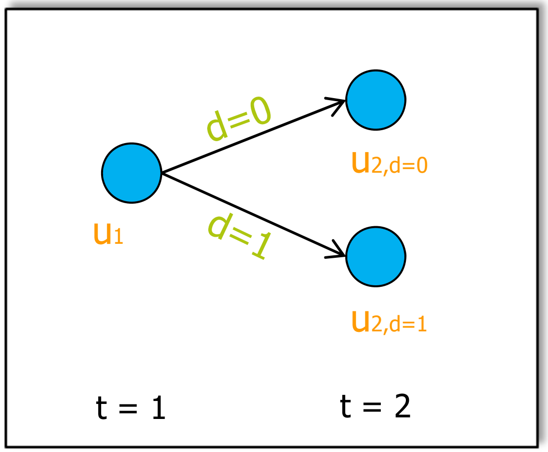

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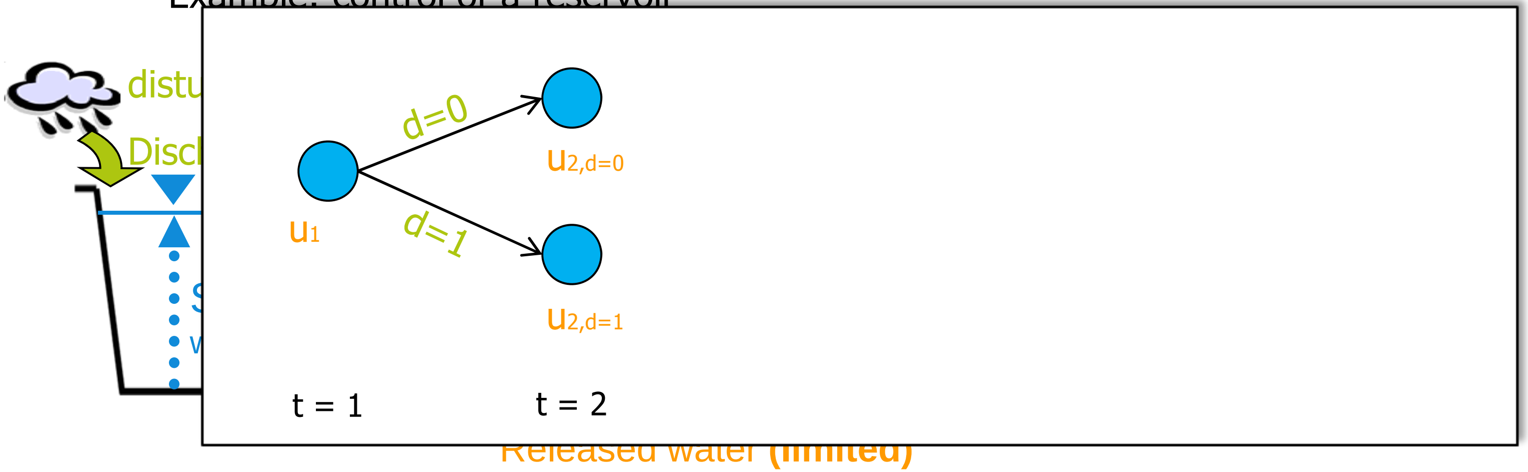

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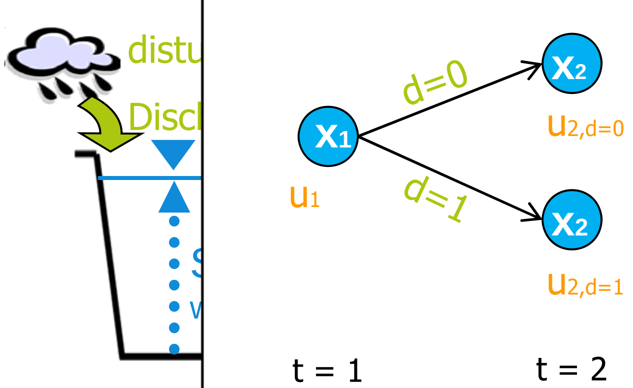

Decision variables:

$$\left\lfloor u_1, u_{2|d=0}, u_{2|d=1} \right\rfloor$$

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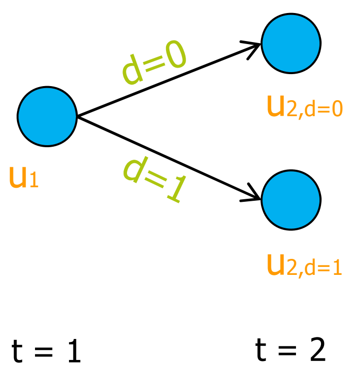

Released water **(limited)**

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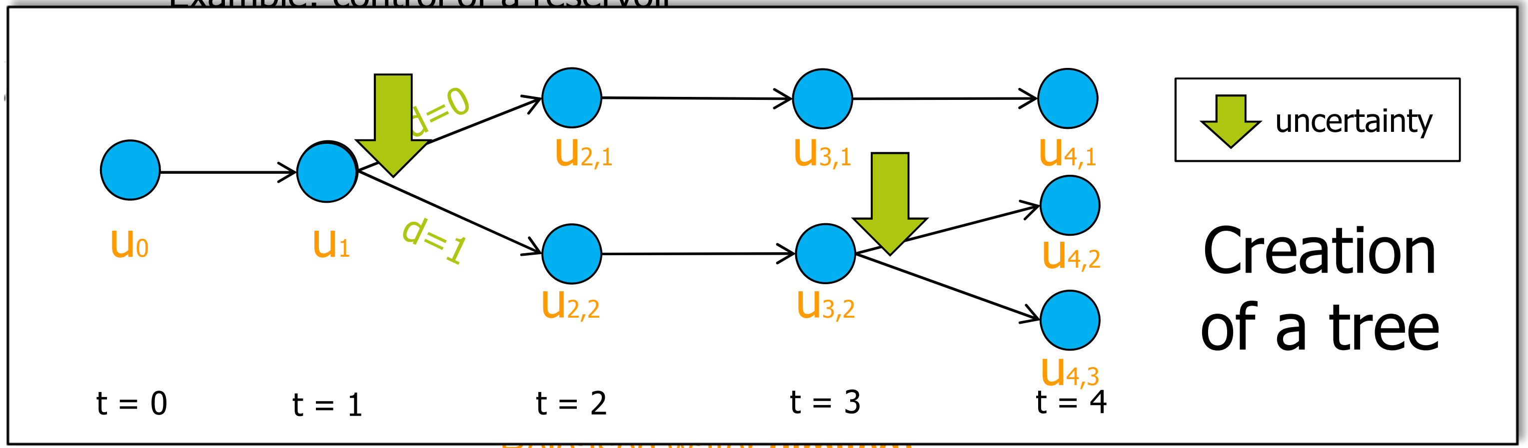

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Release at t_1
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Release at t_2
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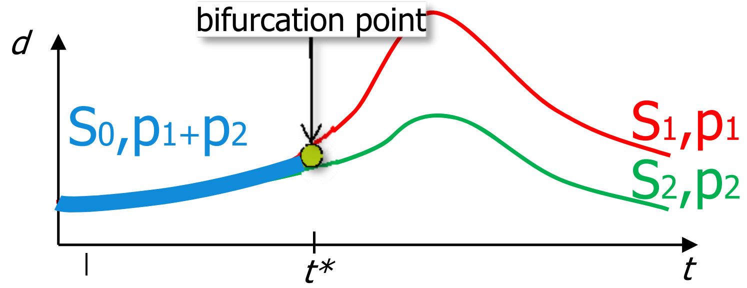
How to create a tree from an ensemble

Existing procedures for scenario aggregation

- Existing procedure: "*optimal scenario reduction*" technique
  - Aggregate scenarios when "sufficiently close", i.e. average distance on [0,t*] smaller than a fixed threshold



This method requires the definition of

- Distance
- Threshold

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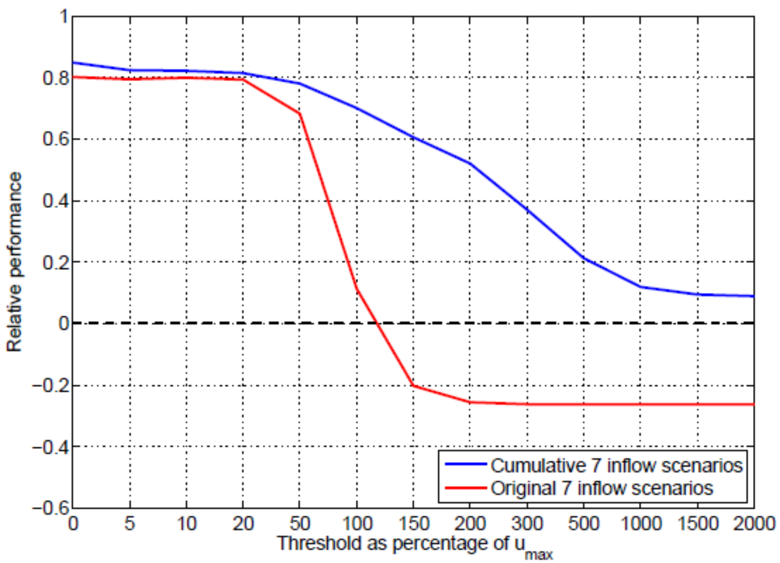
Threshold value sensitivity analysis



Optimal control of the reservoir using a tree

Relative performance index:
1: perfect forecast, no uncertainty
0: deterministic forecast

Importance of placing the bifurcation points a the right moment

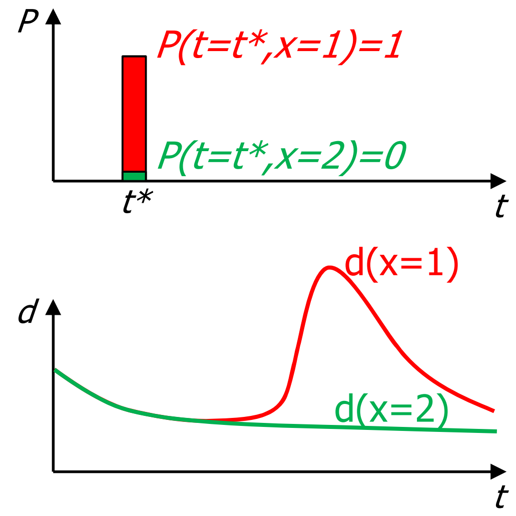
- Threshold

How to create a tree from an ensemble

An Information Based approach

- Bifurcation points are to be placed as soon as the uncertainty is solved, i.e. where probability of occurrence is sufficiently close to one

- Example: stream response with delay
    - Two scenarios:
        - Scenario x=1: rain at *t**
        - Scenario x=2: no rain
    - Observable dimensions:
        - Precipitation *P*
        - Discharge *d*



Rule: find $\min_t p(x) > p^*$

How does information enter the system?

Update of scenarios probability

- Observable variables $y(t, x, j)$
  - t = time *[1,…,h]*
  - x = scenario *x ϵ X:{1,2,…,N}*
  - j = observable dimension *discharge, rain, etc,…*

$y_t \mid x$ is a random variable f_y distributed

$$y_t = \hat{y}_t + \varepsilon \qquad \varepsilon \sim N\left(0, \sigma^2\right)$$

- Use of the Bayesian theorem:

$$p(x|\hat{y}_t) = \frac{f_y(\hat{y}_t|x) \cdot p(x)}{\sum_i^N f_y(\hat{y}_t|x_i) \cdot p(x_i)}$$

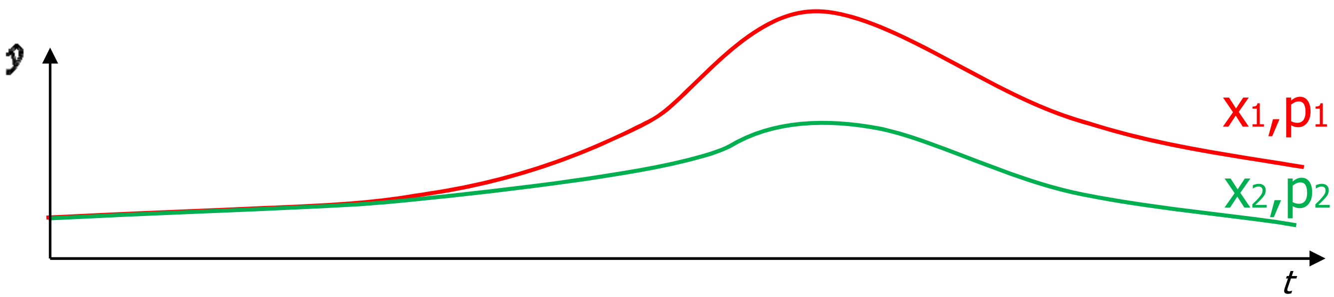
Update of the probability $p(x)$ given the observation $\hat{y}_t$

How does information enter the system?

Update of scenarios probability

- Observable variables $y(t, x, i)$

- Two scenarios $[x_1, x_2]$ with *a priori* probability $[p_1, p_2]$
- Observable dimension: $\hat{y}$ (present discharge)



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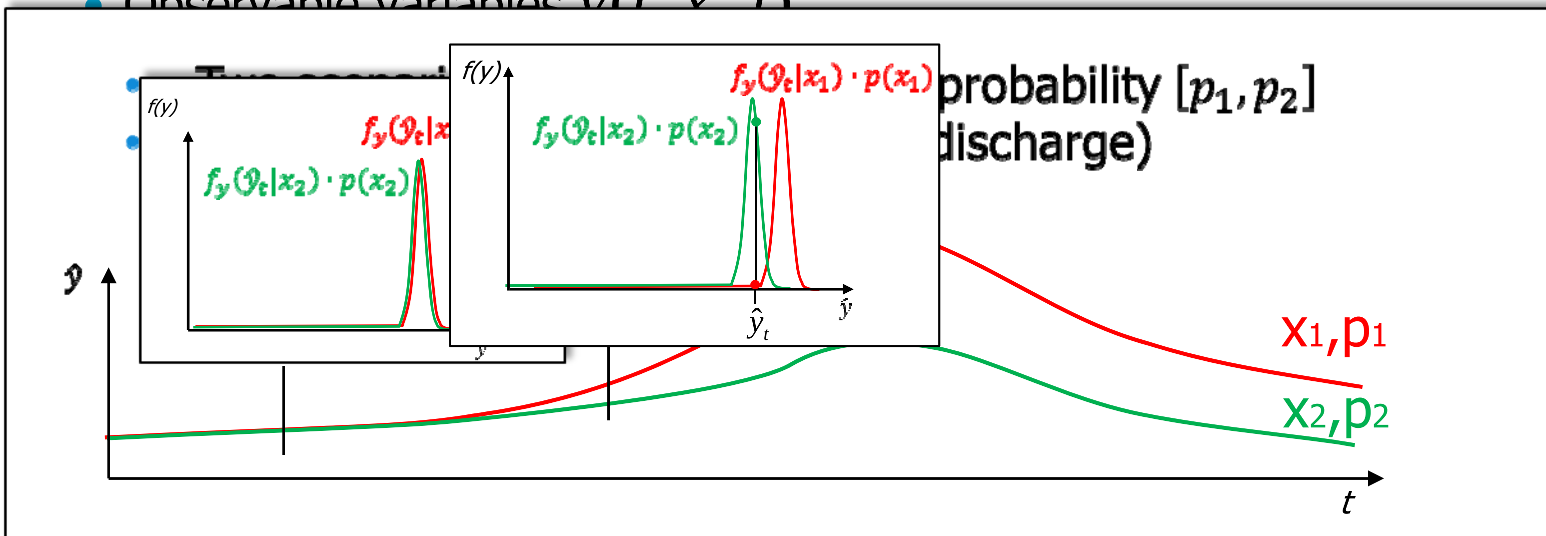

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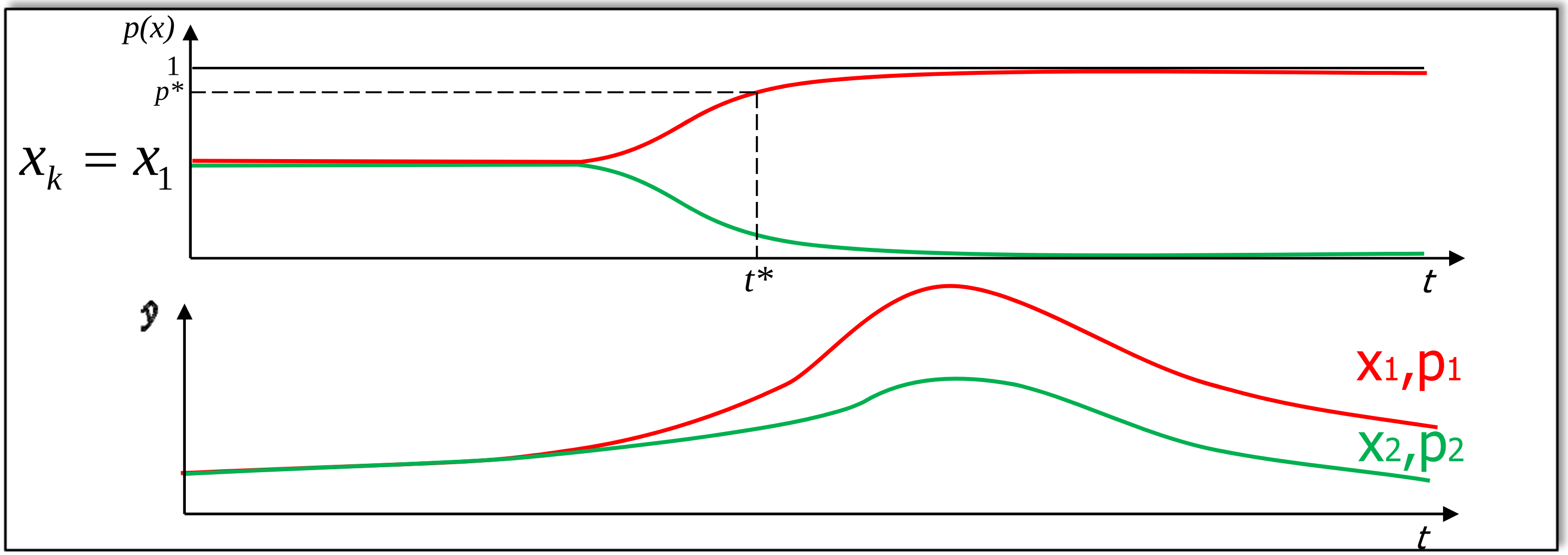
Update of the probability $p(x)$ given the average observation $\hat{y}_t|x_k$

How does information enter the system?

Update of scenarios probability



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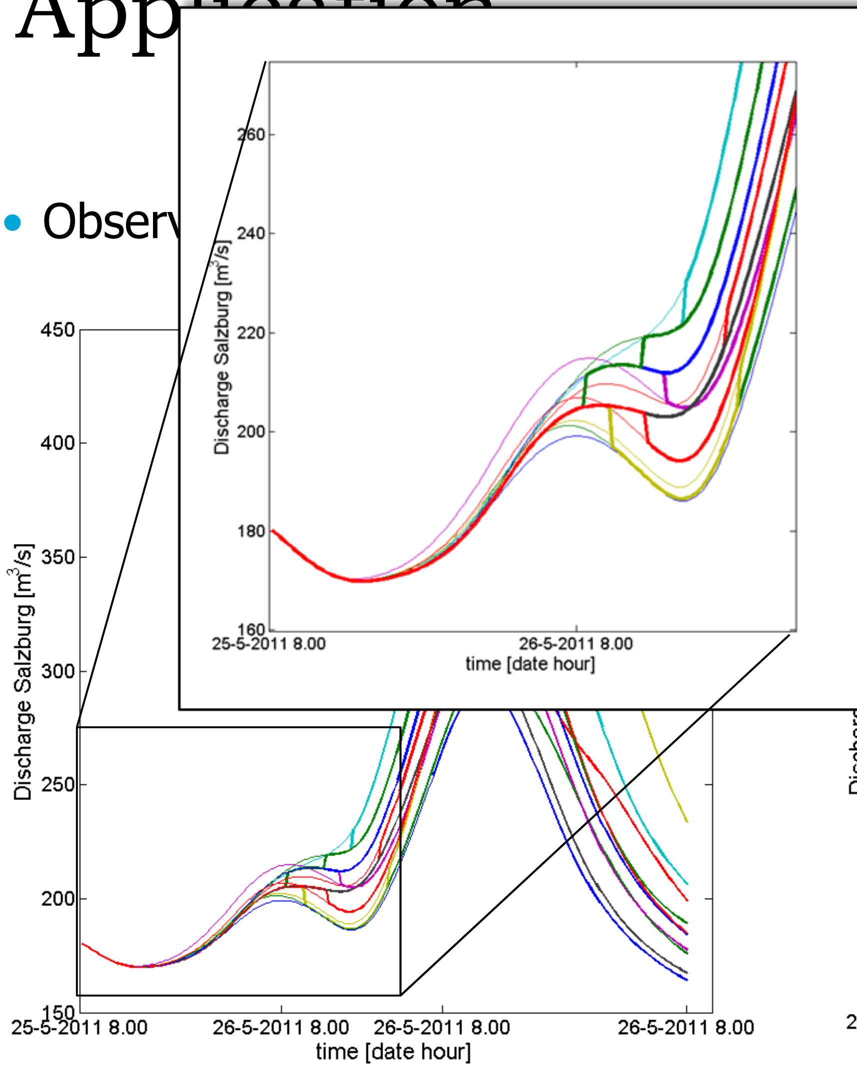
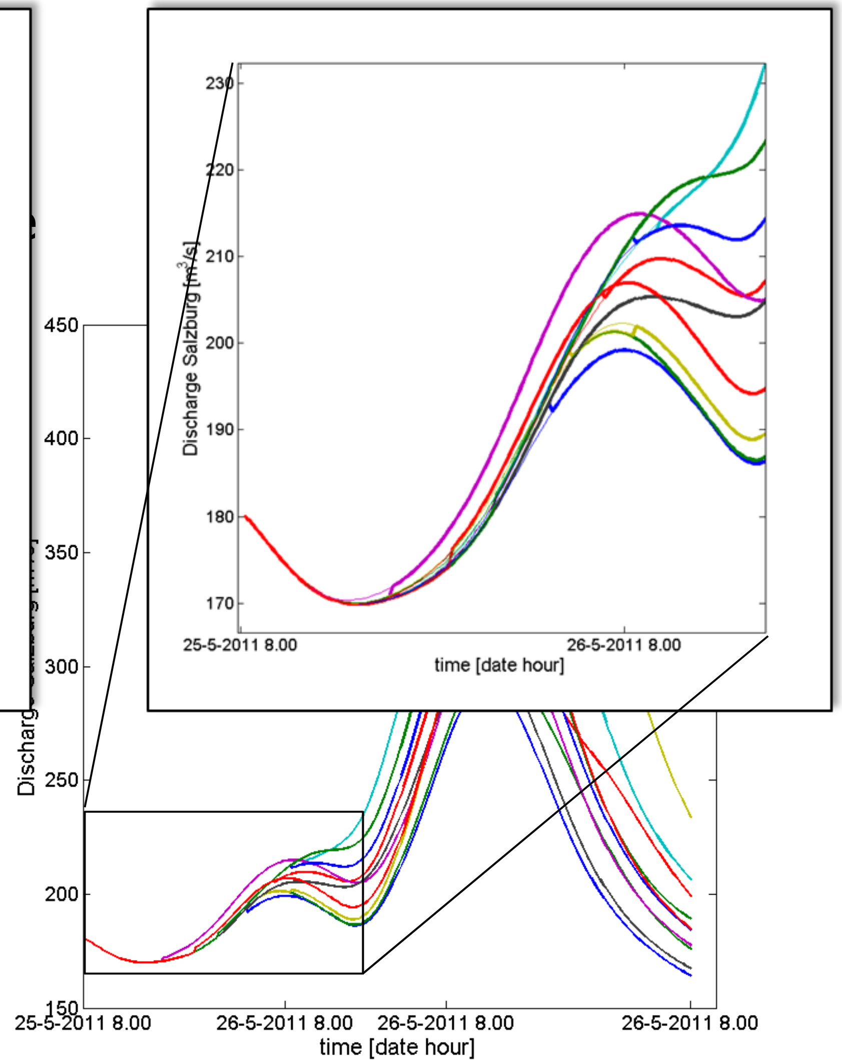
Application

- Observ



$\sigma = 5\ m^3/s$



$\sigma = 1\ m^3/s$

Future application

- Observable variable: discharge forecast

$$Y_t \mid x \sim N(\mu, \Sigma)$$

$$Y_t = \left[y_{t|t}, y_{t+1|t}, \ldots, y_{t+h|t}\right]$$
$$\mu = \left[E\left(y_{t|t} \mid x\right), E\left(y_{t+1|t} \mid x\right), \ldots, E\left(y_{t+h|t} \mid x\right)\right]$$
$$\Sigma = COV\left(y_{t+i|t} \mid x, y_{t+j|t} \mid x\right)_{i=1,2,\ldots,h;\, j=1,2,\ldots,h}$$

Comments and conclusion

- In the cases when uncertainty is solved in time

  A tree is a <u>better model of uncertainty</u> than an ensemble

- It is possible to generate a tree from an ensemble
  - the “Information Based” method requires the definition how information enters the system

Question Time

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