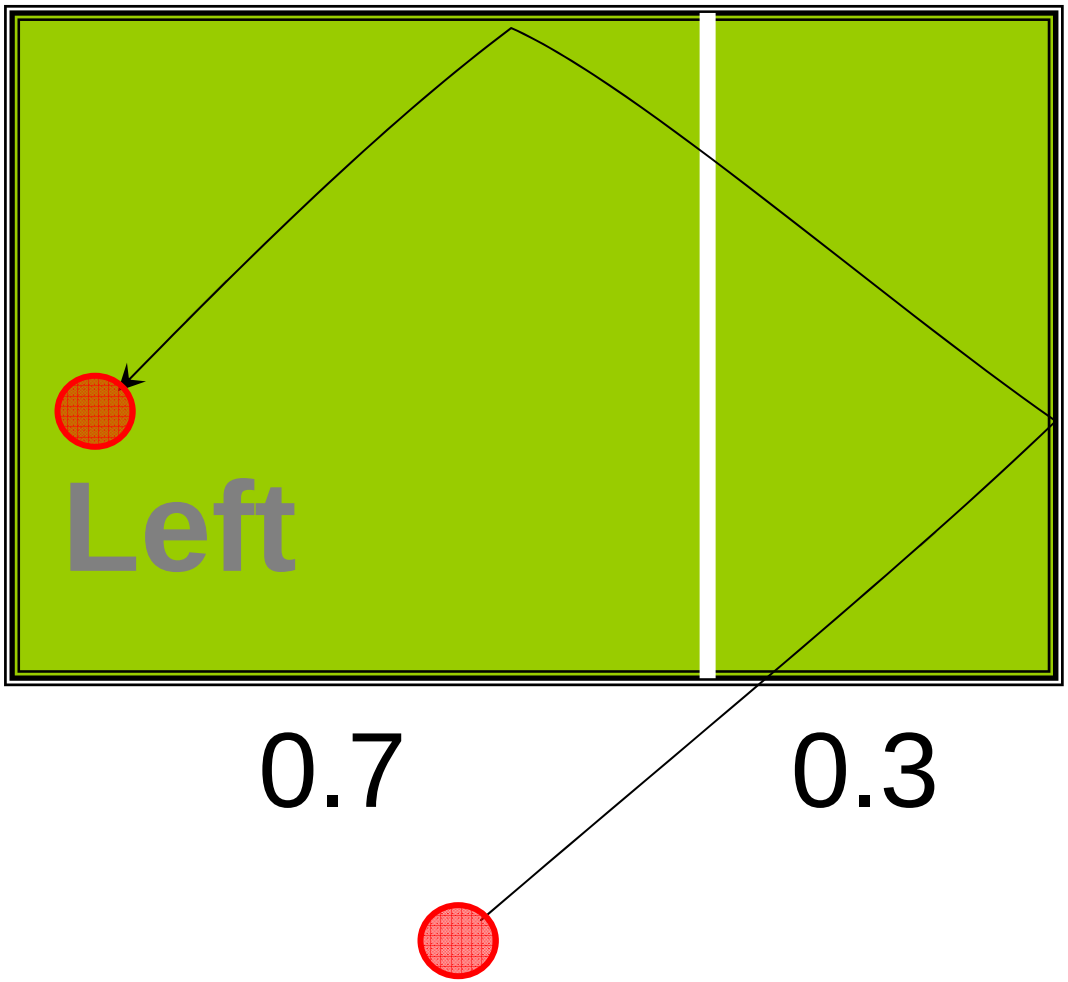
III Subjective probabilities

2. Bayesianism according to Bayes

III.2.1 Bayes's billiard table experiment

Bayes's experiment as it would have been set up by (de Moivre) a classicist:



Throwing 3 balls

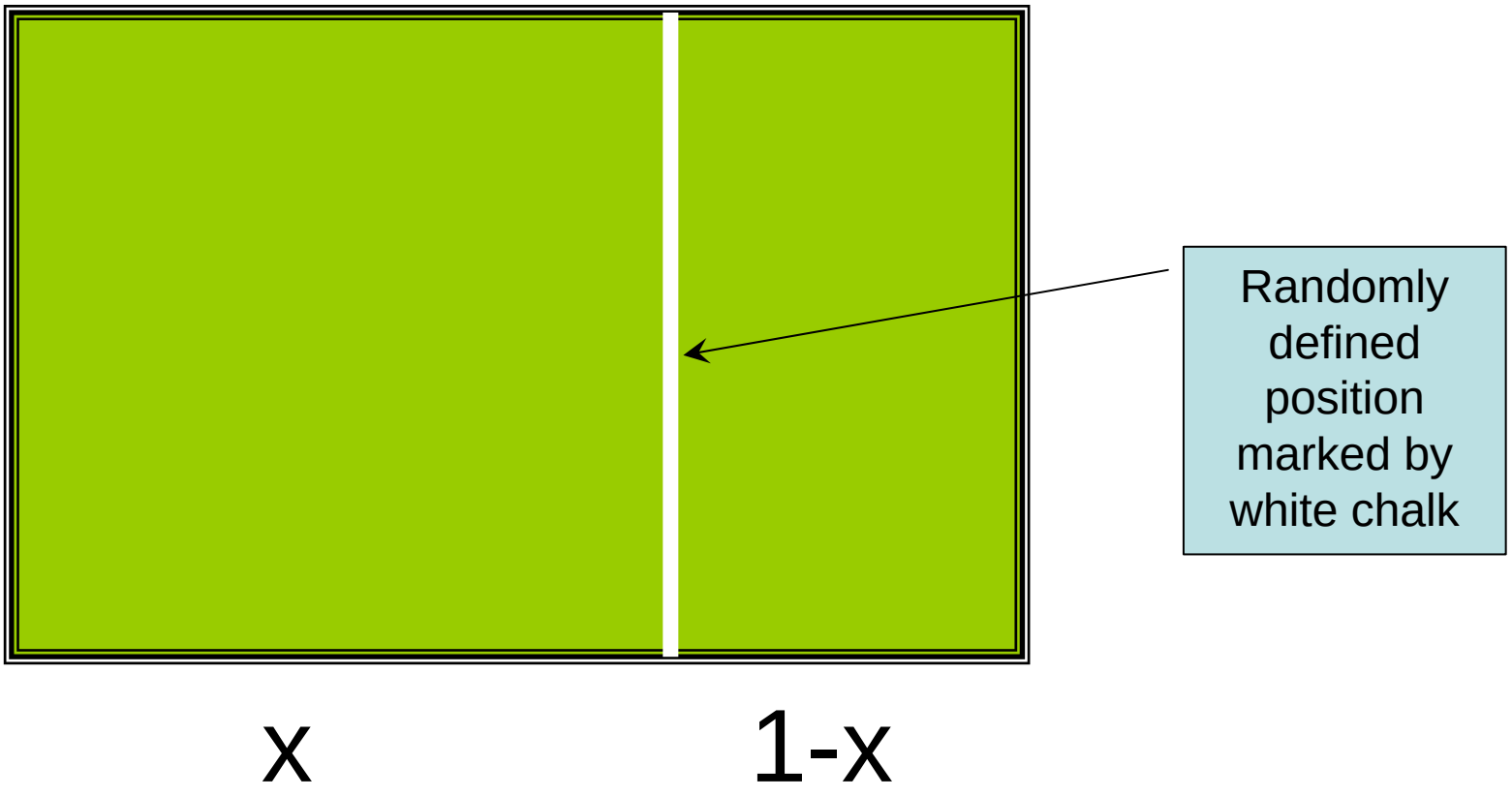
Prob(RRR)= 3%

Prob(RRL,RLR,LRR) =19%

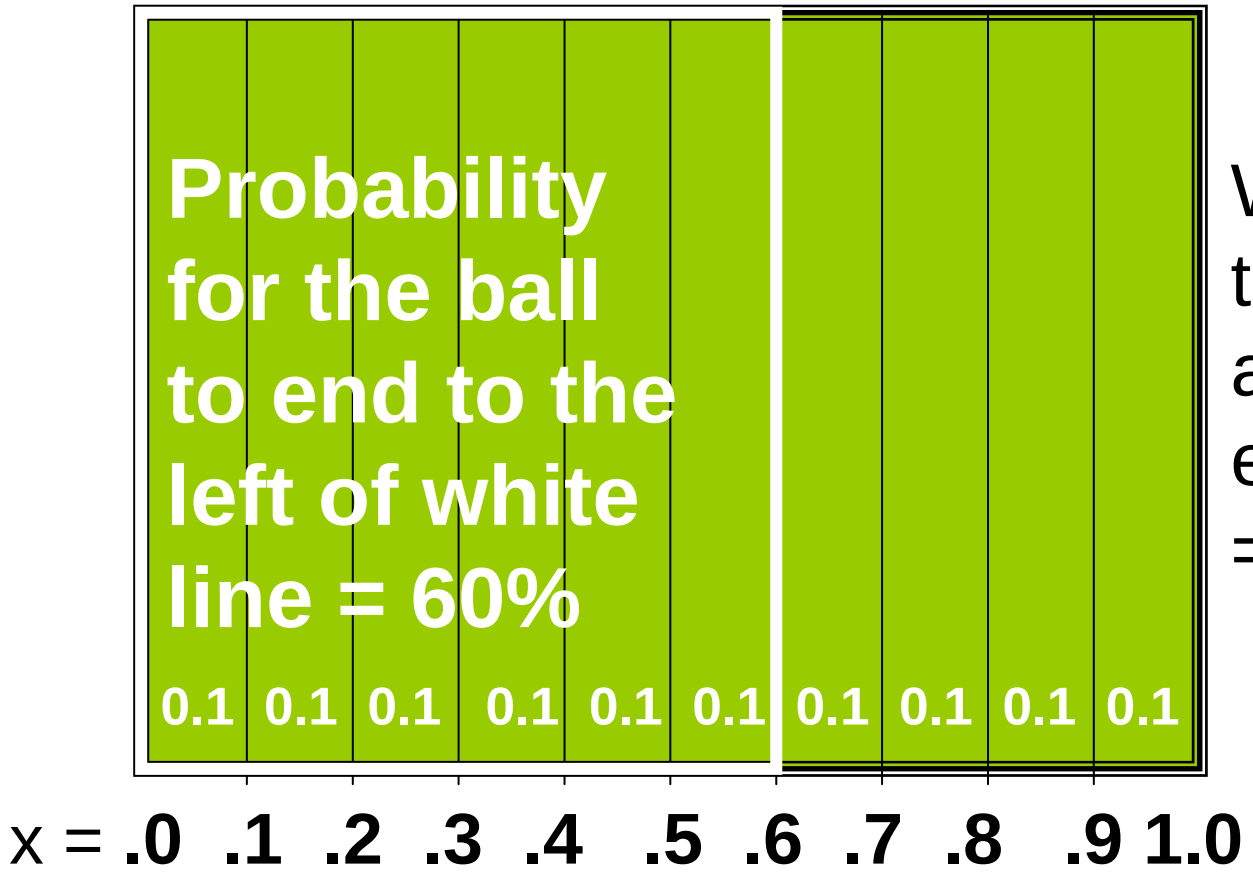
Prob(RLL,LRL,LLR)= 44%

Prob(LLL)=34%

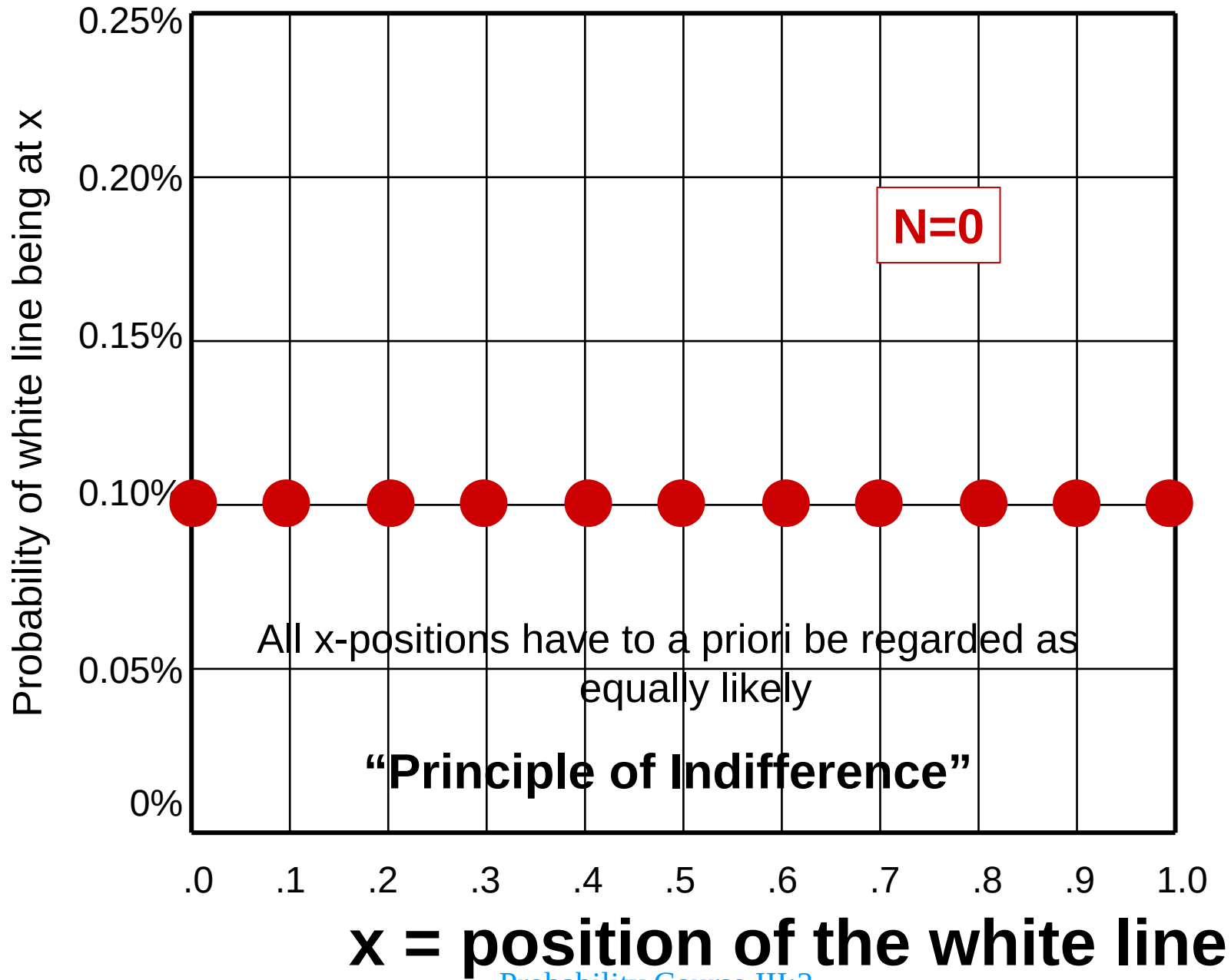
Thomas Bayes' experiment



The length of the table is divided into 10 sections



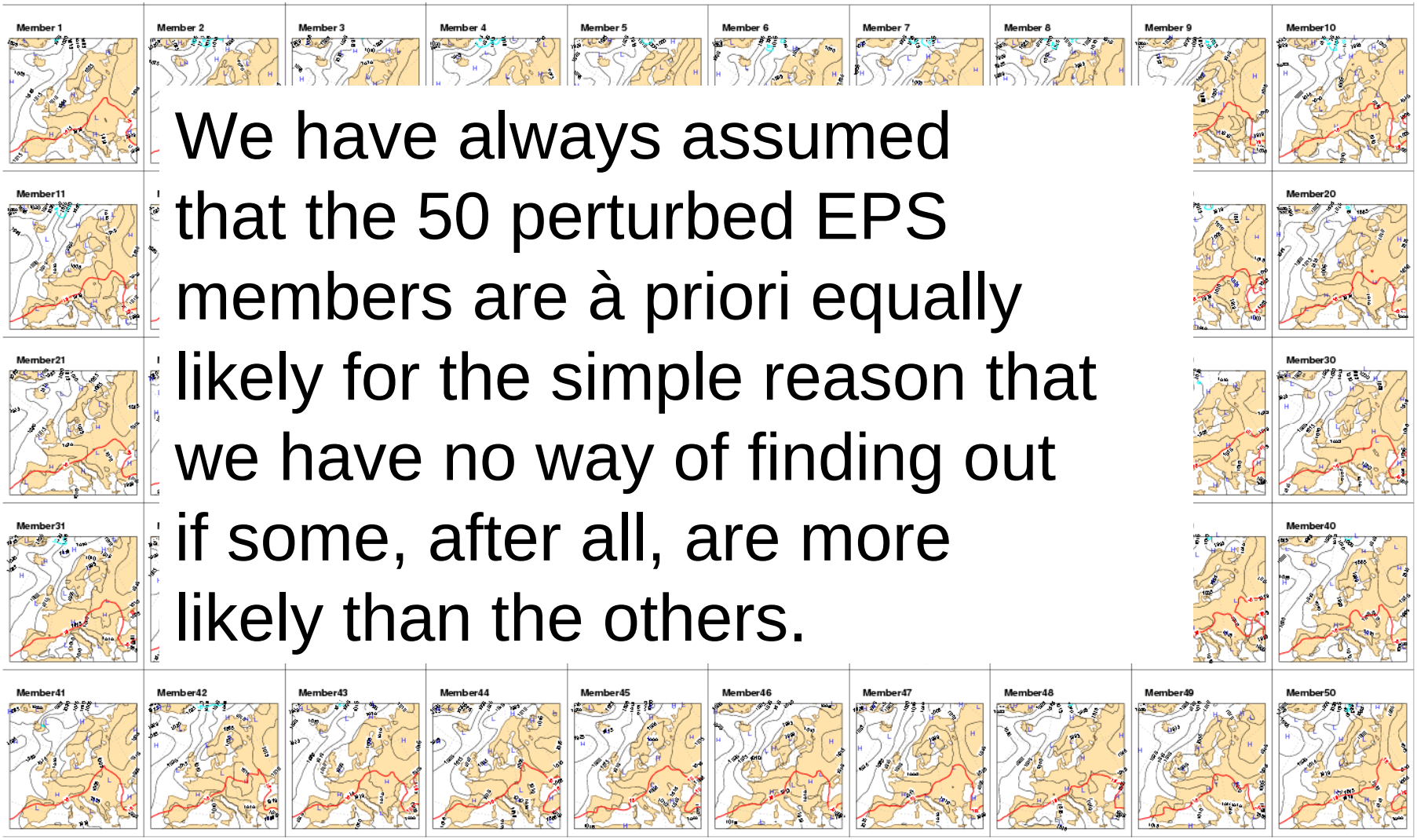
We assume that the ball can take any position with equal probability = 10%


Probability of white line being at x
0.25%
0.20%
0.15%
0.10%
0.05%
0%
N=0
All x-positions have to a priori be regarded as equally likely
“Principle of Indifference”
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
x = position of the white line

III.2.3 “The Principle of Indifference”

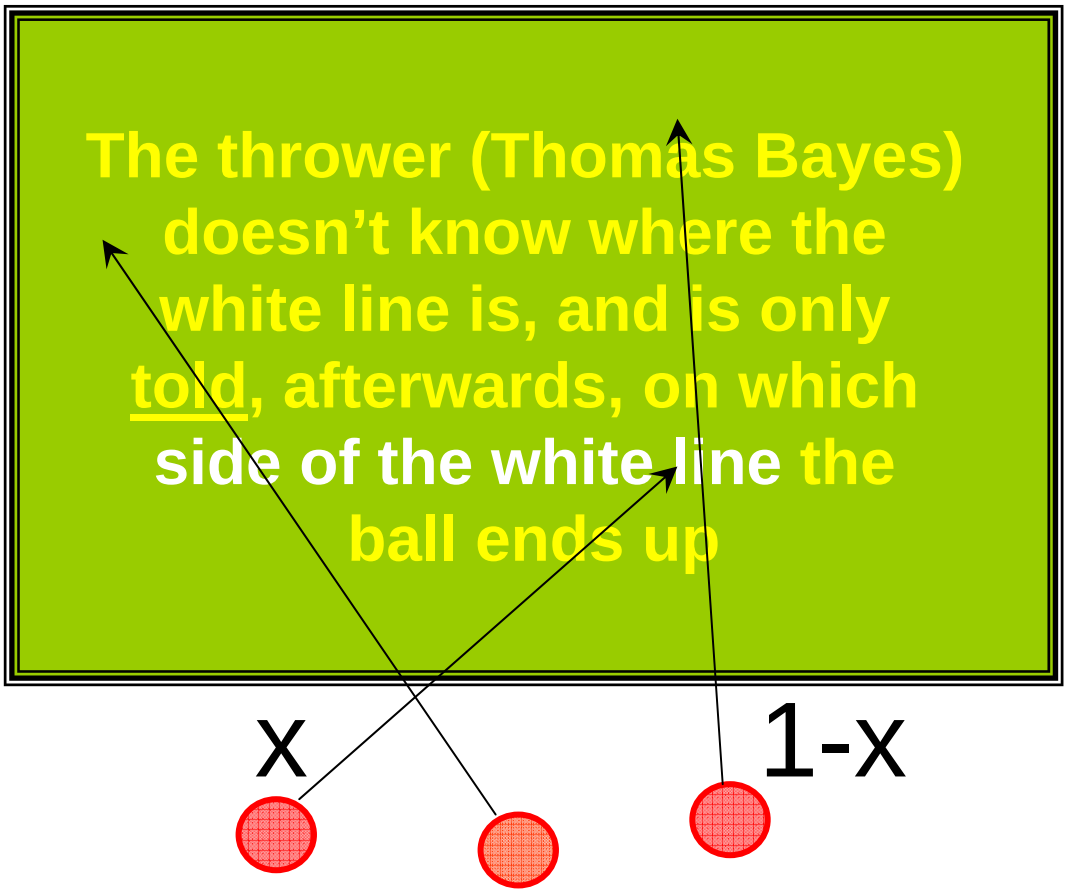
“ “When you do not know the probabilities you light-heartedly assume they are equal!”

“The Principle of Indifference” applied on the EPS?



We have always assumed that the 50 perturbed EPS members are à priori equally likely for the simple reason that we have no way of finding out if some, after all, are more likely than the others.

Thomas Bayes' experiment



Left

Left

Right

Let A_i be x=0.4 and B = ball to the left

$$p(A_i|B) = \frac{p(A_i)\cdot p(B|A_i)}{\sum_i p(A_i)\cdot p(B|A_i)}$$

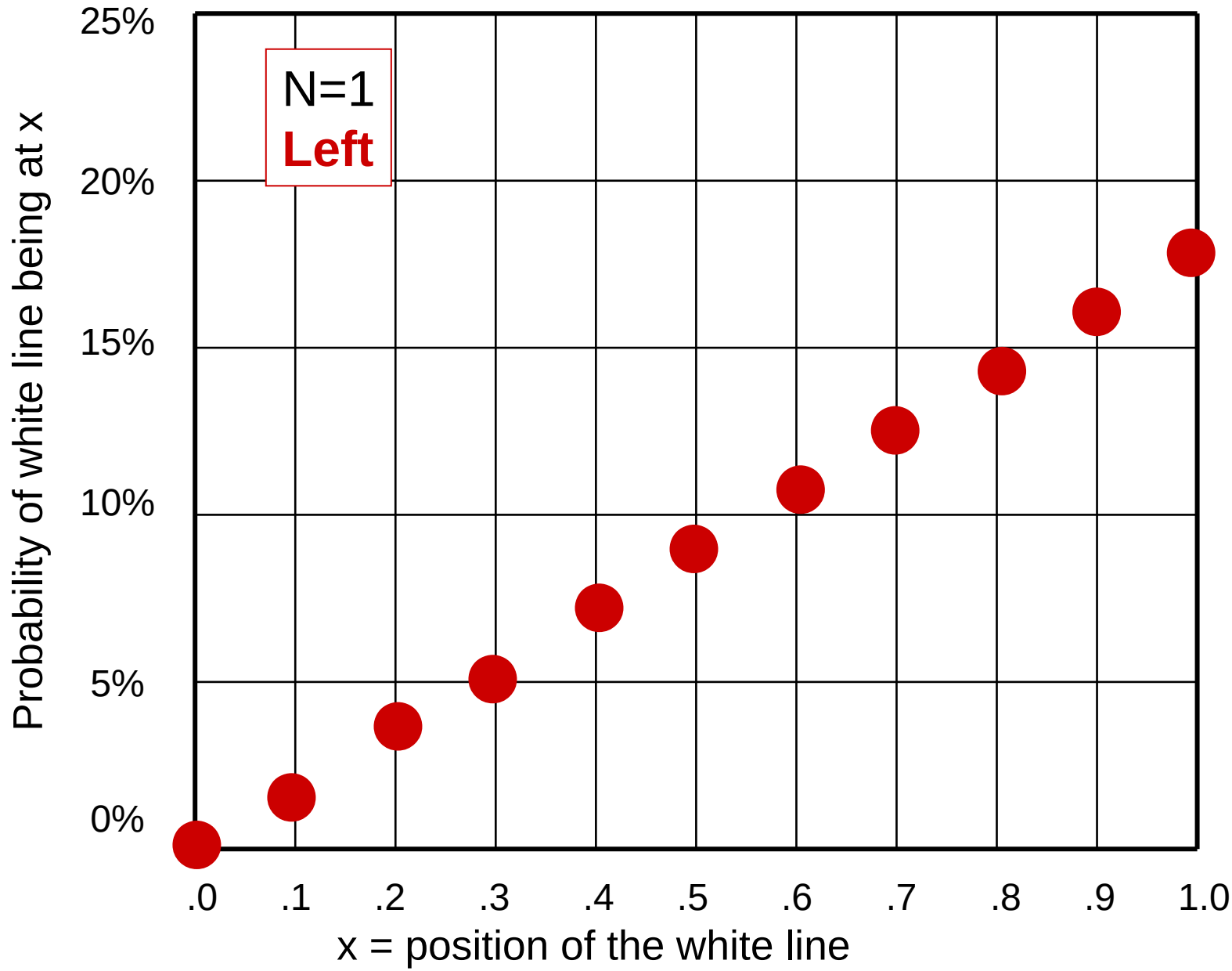
0.1 as initially assumed

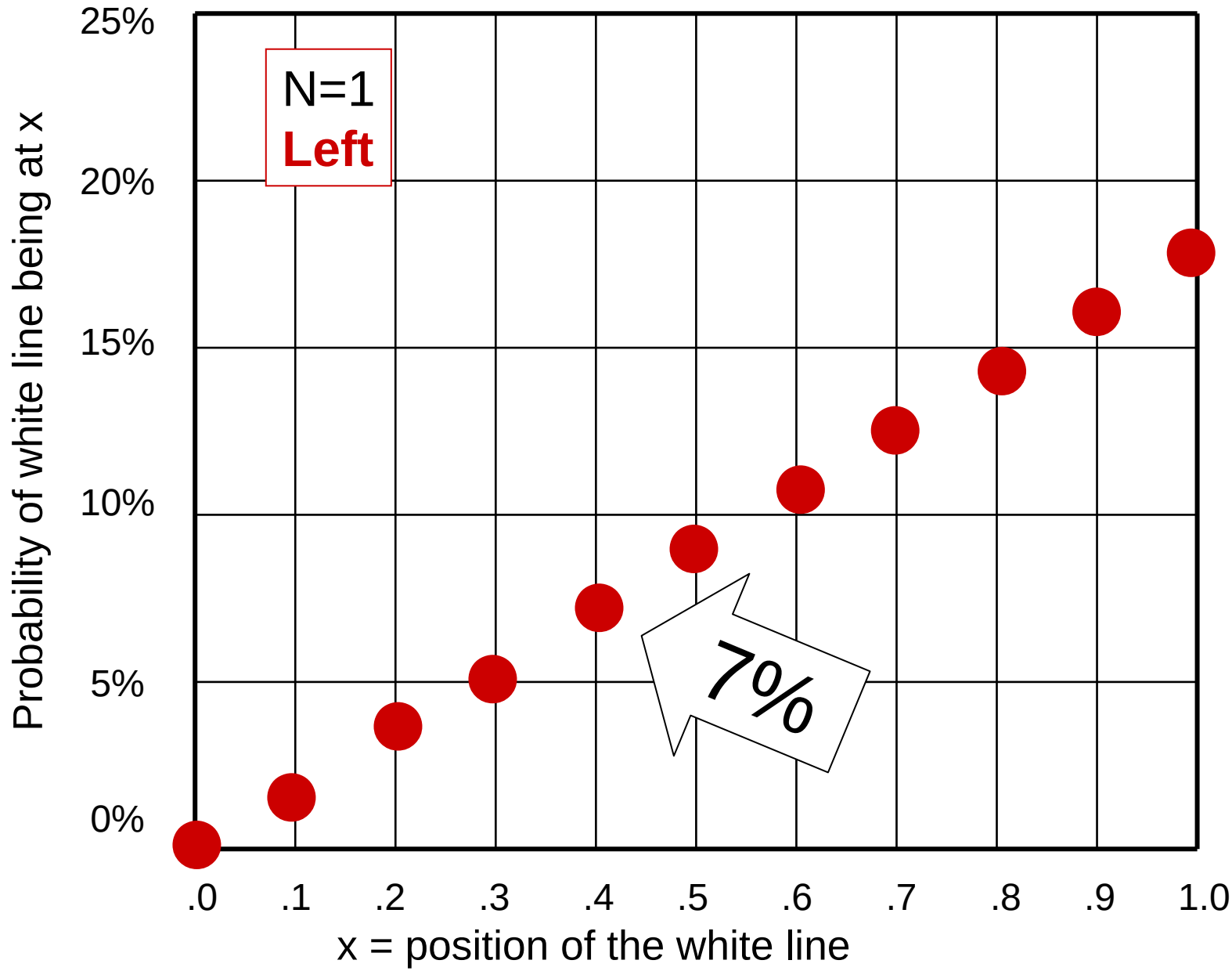
0.4 per definition

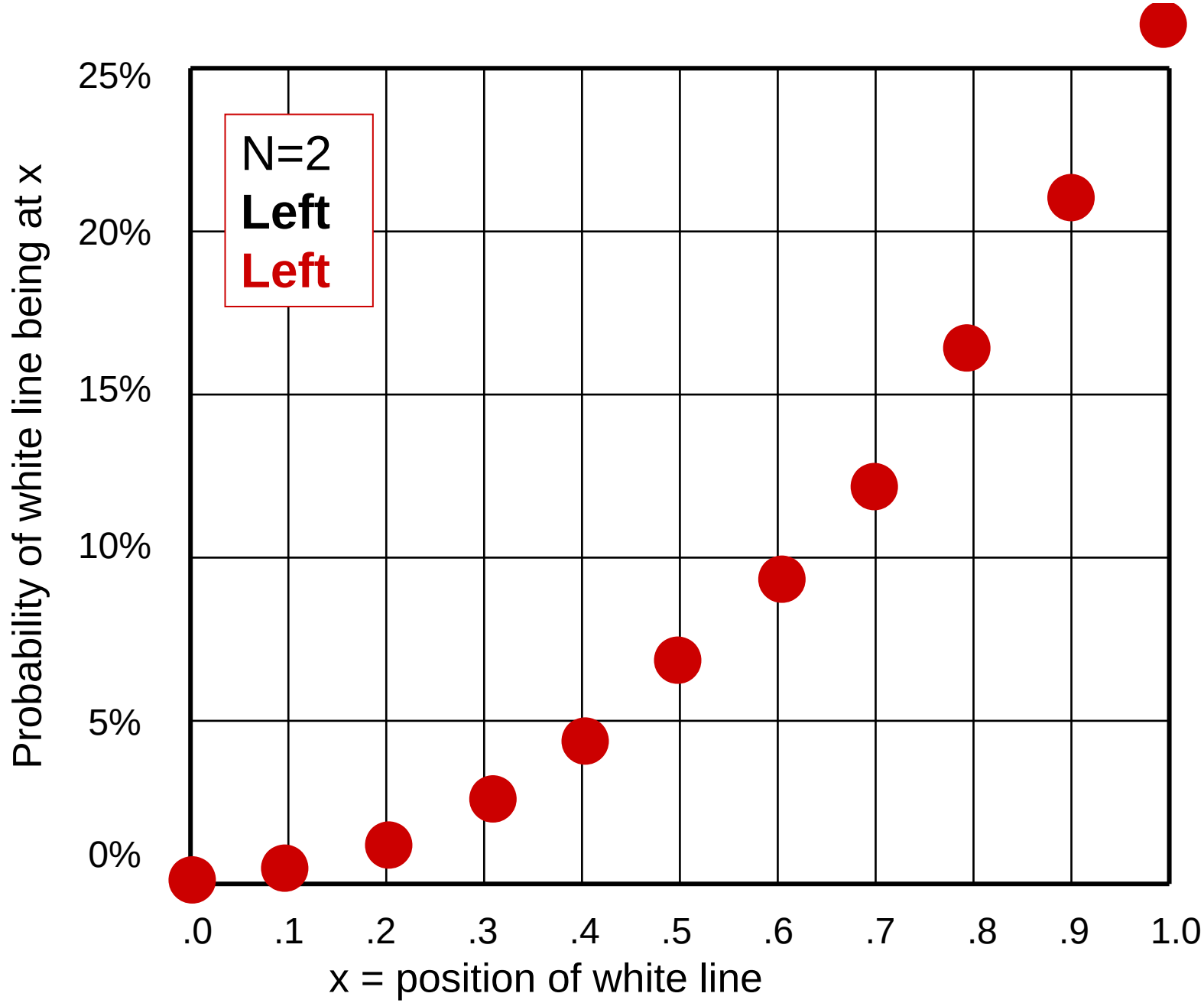
$$\mathbf{p(x=0.4|left)} = \frac{p(x=0.4)\cdot p(left|x=0.4)}{\Sigma p(all\ x)\cdot p(left|all\ x)}$$

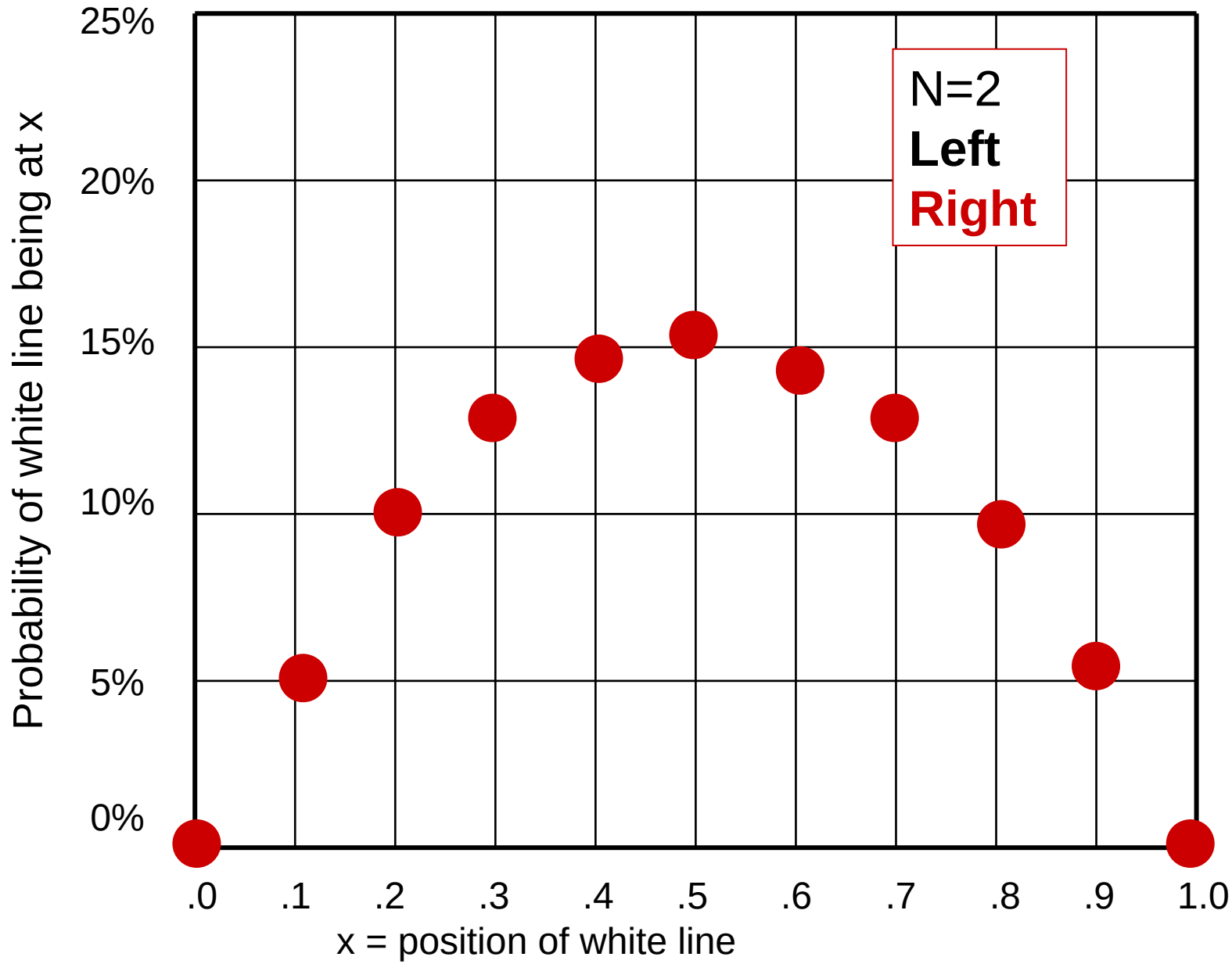
Update of prob(x=0.4) when ball falls to the right 0.1·0.4 / 0.55 = 0.07

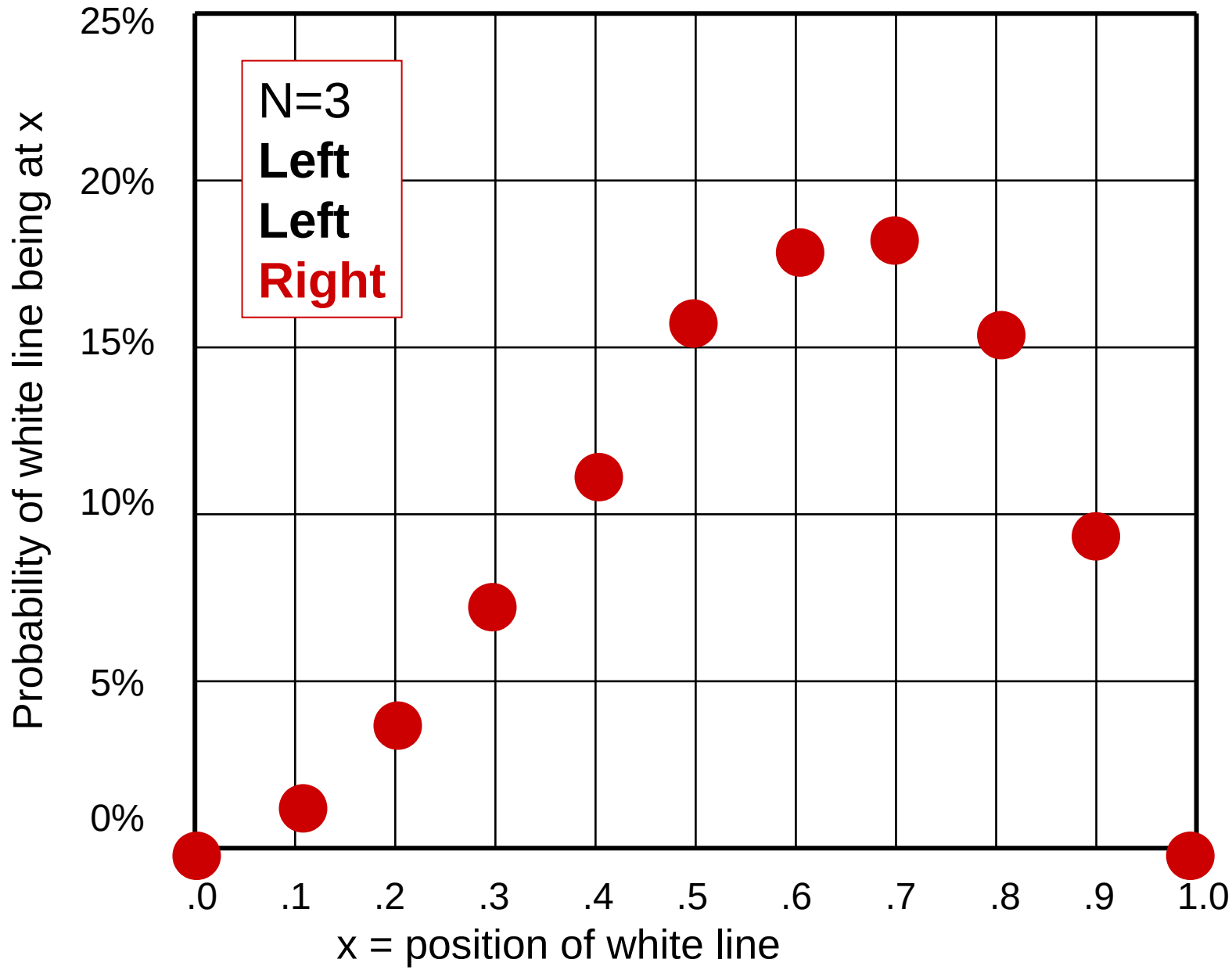
0.1·0.0+0.1·0.1+0.1·0.2+….+0.1·1.0=
0.1·(0.1+0.2+0.2+…+1.0) = **0.55**

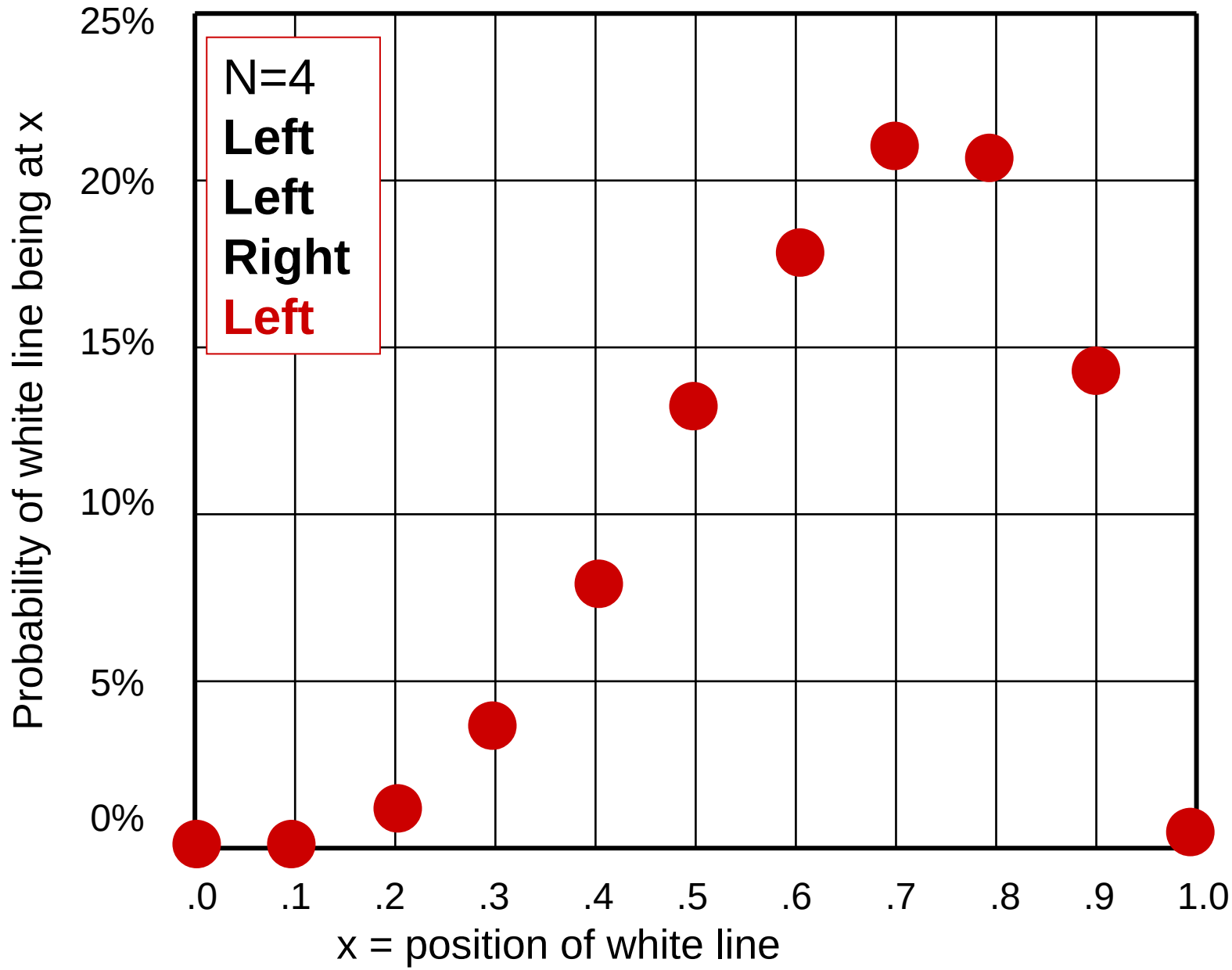

N=1
Left
25%
20%
15%
10%
5%
0%
Probability of white line being at x
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
x = position of the white line


25%
20%
15%
10%
5%
0%
Probability of white line being at x
N=1
Left
7%
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
x = position of the white line

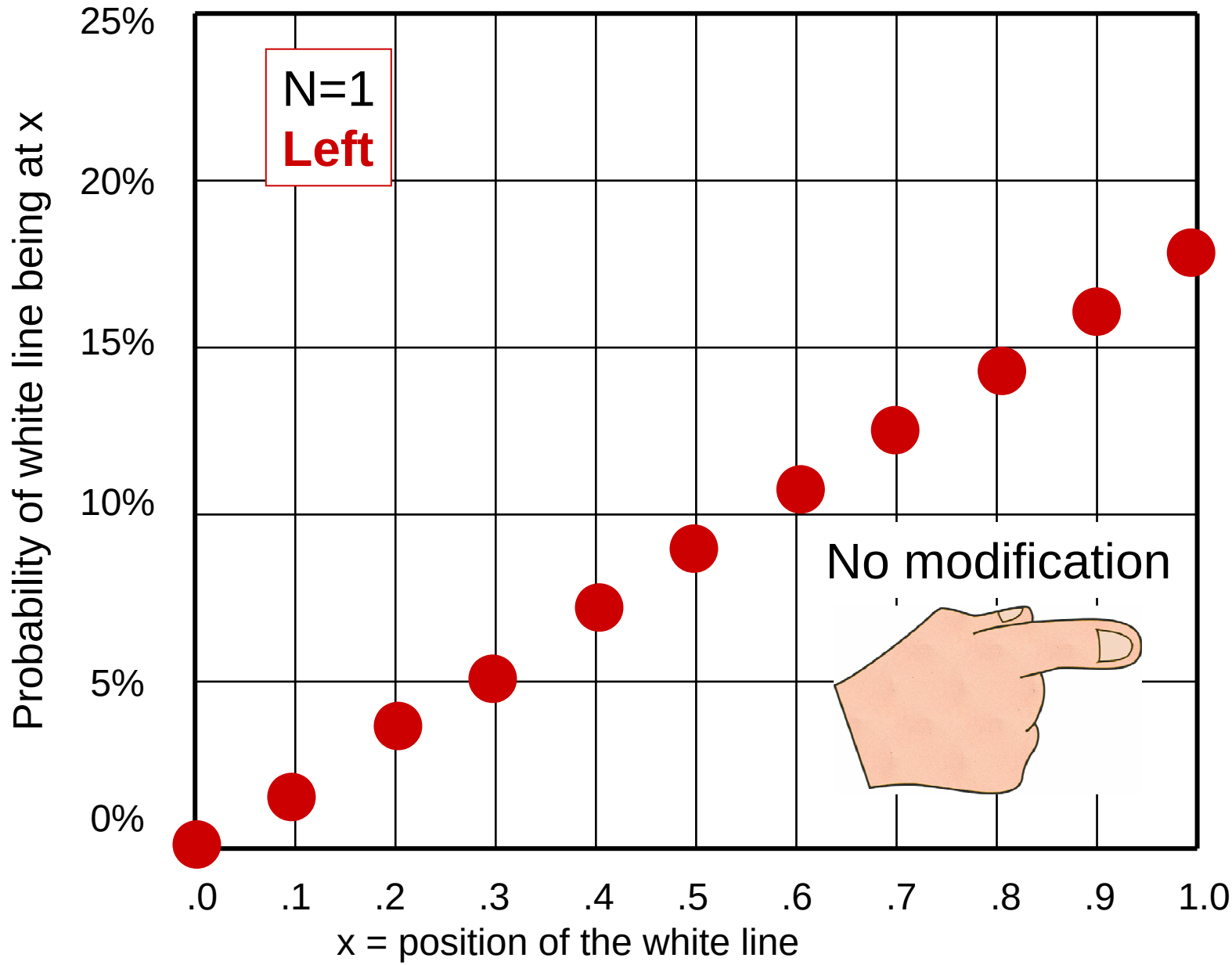

25%
20%
15%
10%
5%
0%
Probability of white line being at x
N=2
Left
Left
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
x = position of white line

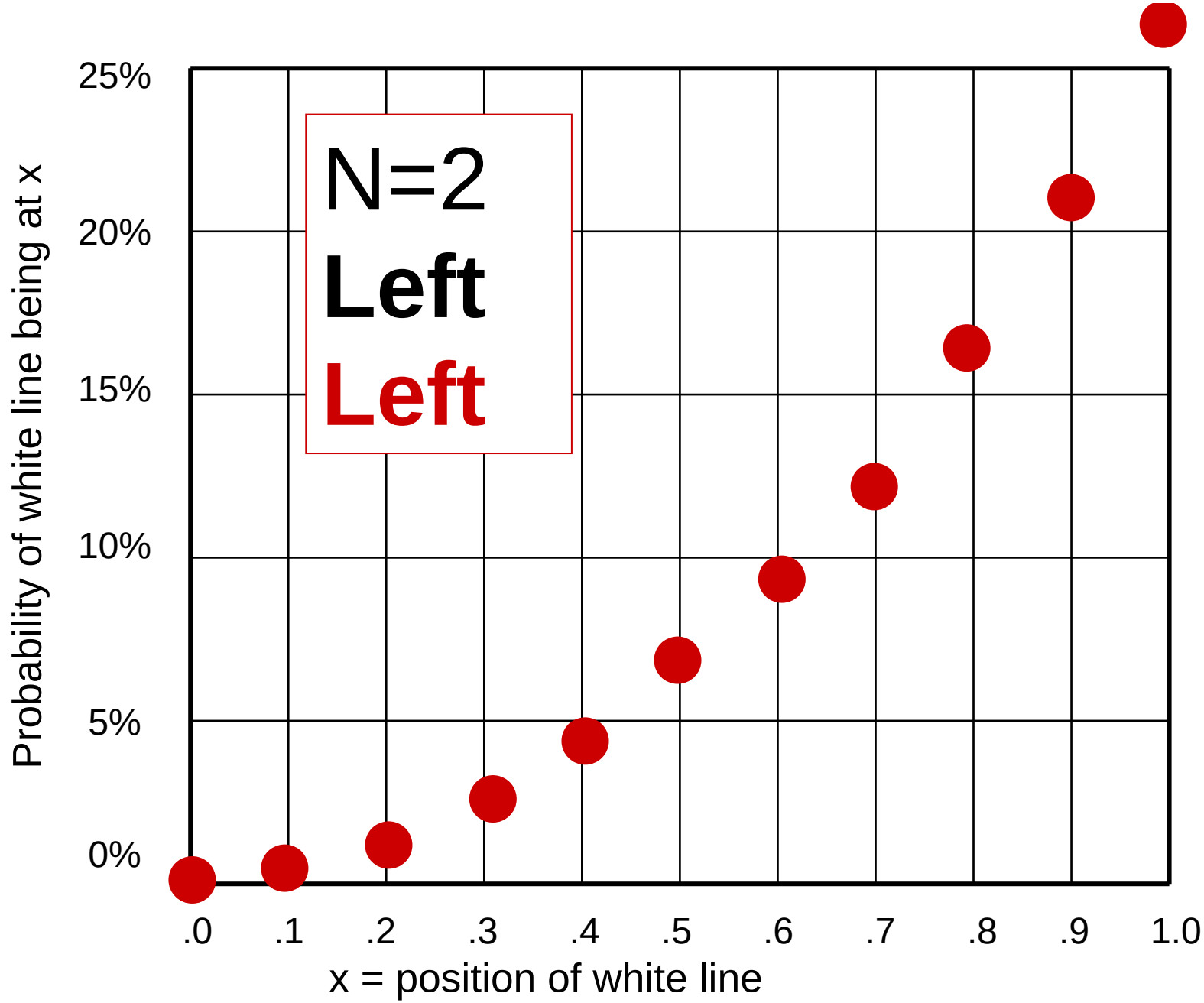

25%
20%
15%
10%
5%
0%
Probability of white line being at x
N=2
Left
Right
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
x = position of white line

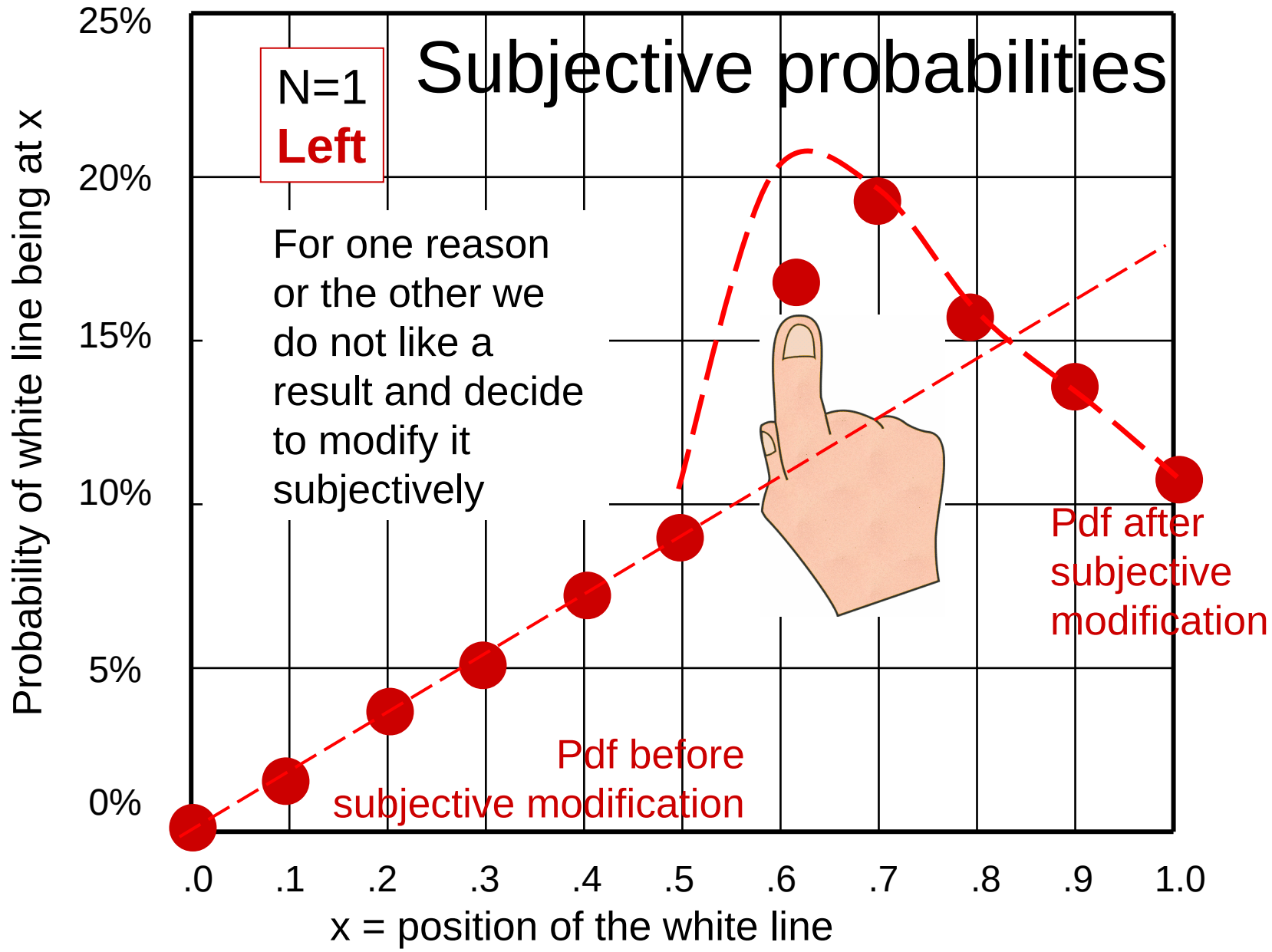

25%
20%
15%
10%
5%
0%
Probability of white line being at x
N=3
Left
Left
Right
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
x = position of white line

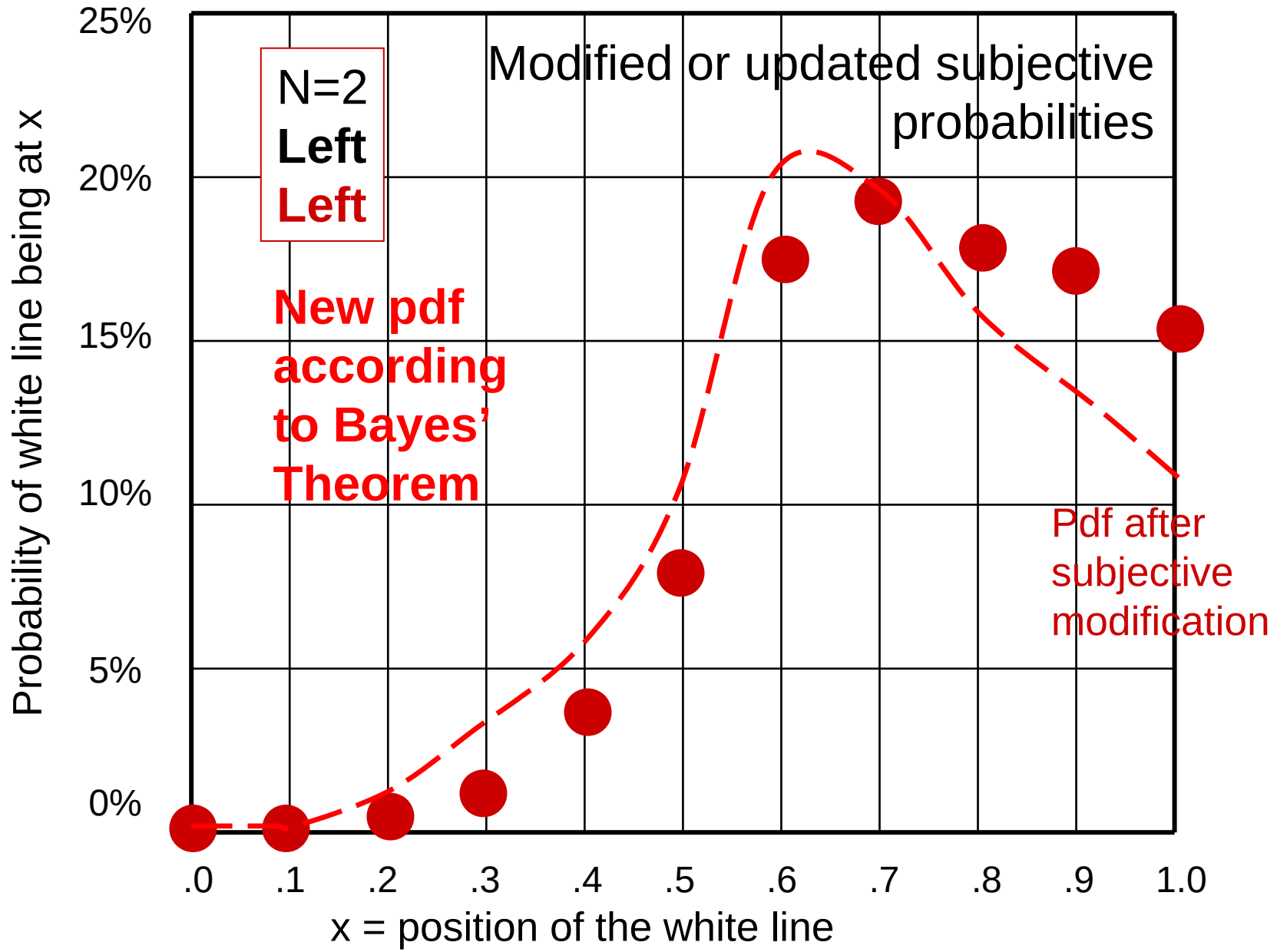

N=4
Left
Left
Right
Left
25%
20%
15%
10%
5%
0%
Probability of white line being at x
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
x = position of white line

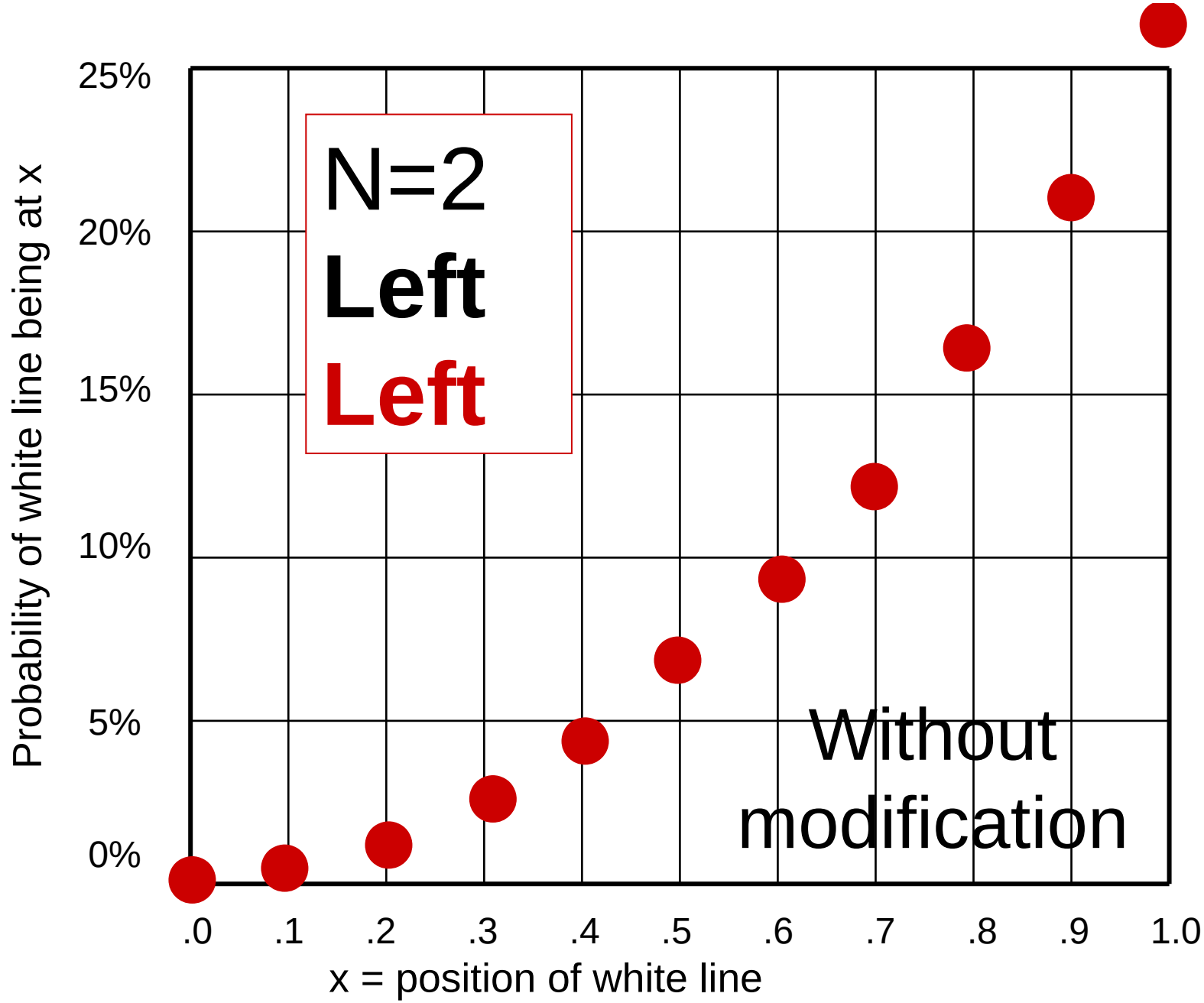
III.2.4 Updating of subjective probabilities


25%
20%
15%
10%
5%
0%
Probability of white line being at x
N=1
Left
No modification
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
x = position of the white line


N=2
Left
Left
Probability of white line being at x
25%
20%
15%
10%
5%
0%
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
x = position of white line

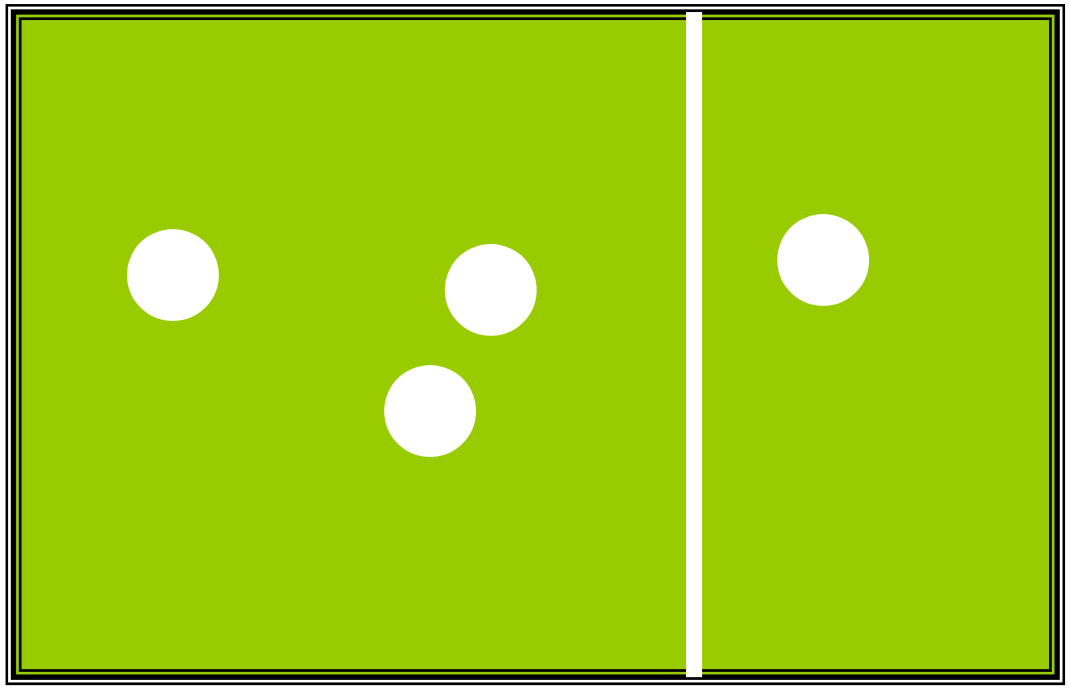

Subjective probabilities
N=1
Left
For one reason or the other we do not like a result and decide to modify it subjectively
Pdf after subjective modification
Pdf before subjective modification
Probability of white line being at x
25%
20%
15%
10%
5%
0%
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
x = position of the white line


25%
20%
15%
10%
5%
0%
Probability of white line being at x
N=2
Left
Left
Modified or updated subjective probabilities
New pdf according to Bayes' Theorem
Pdf after subjective modification
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
x = position of the white line


25%
20%
15%
10%
5%
0%
Probability of white line being at x
N=2
Left
Left
Without
modification
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
x = position of white line

III.2.5 Laplace's Rule of Sucession

After 4 throws 3 are left and 1 right



x 1-x

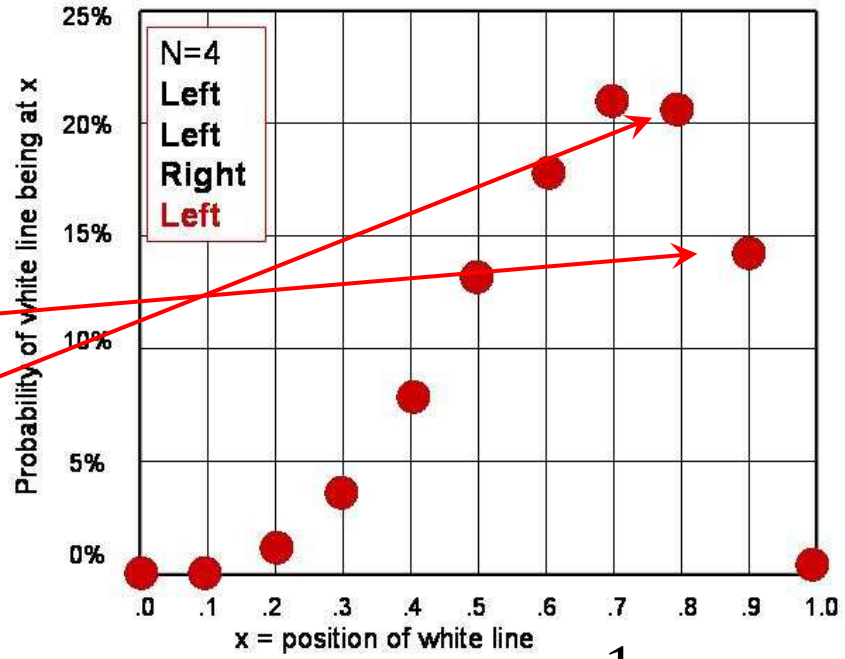
What are the chances of having 2 *subsequent* "left" throws? ***You are invited to bet***

According to the frequentist method the best estimate is

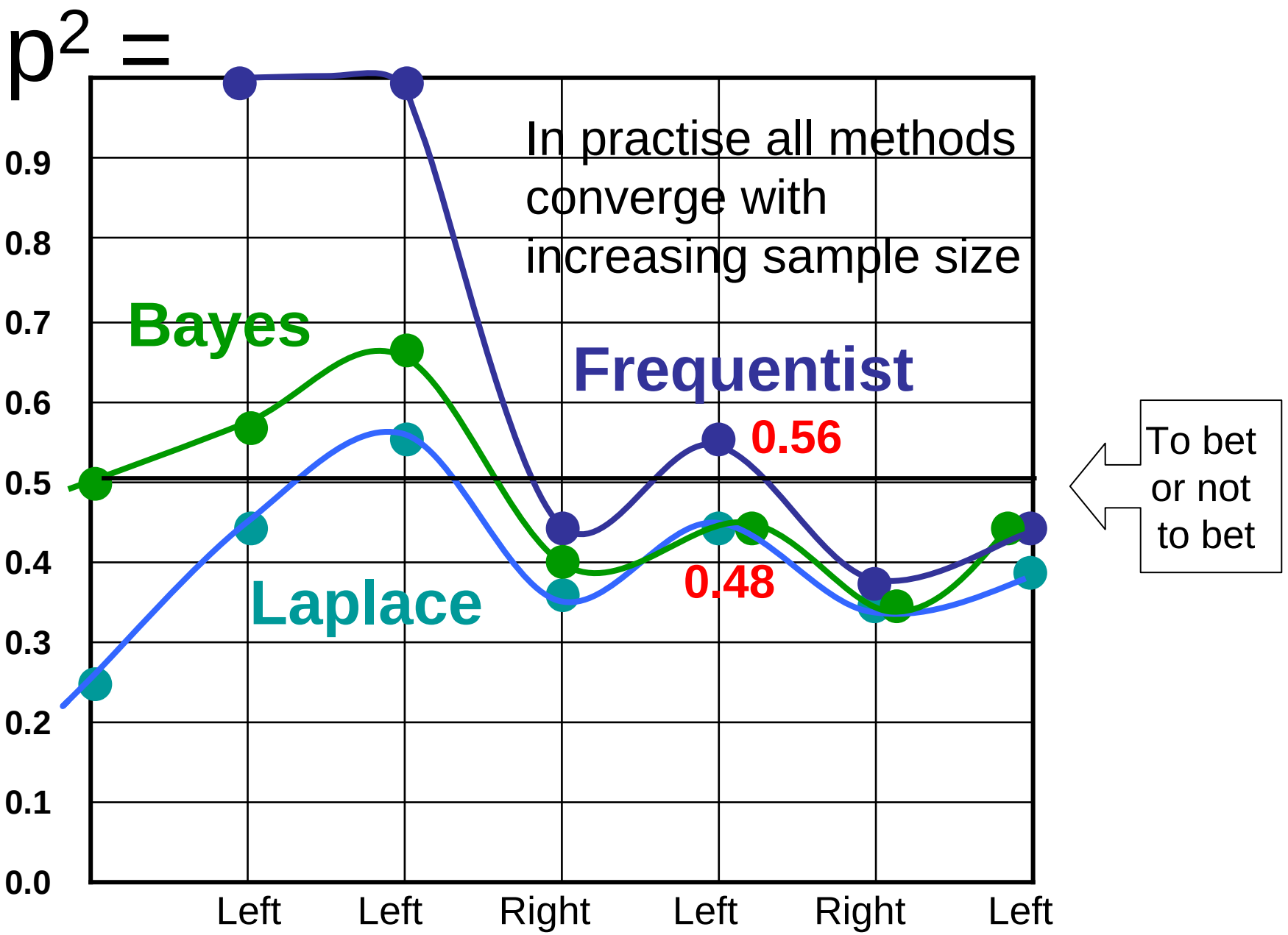
$p^2=(3/4)^2=\mathbf{0.56}$

$$p^2=\left(\frac{N_{Left}}{N}\right)^2$$

According to the Bayesian method (15% chance that $p^2 = 0.81$, 21% $p^2 = 0.64$ etc) the best estimate of $p^2=\mathbf{0.45}$

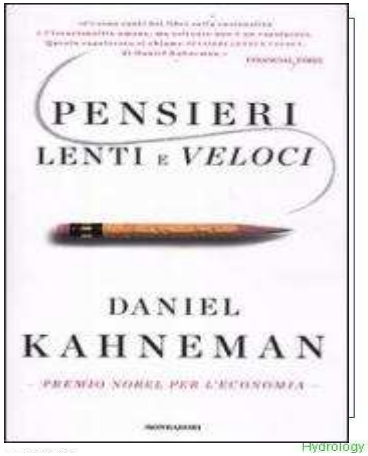


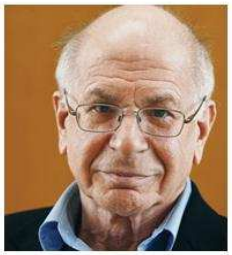
$$p^2=\sum_{x_i=0}^{1} p(x_i)\cdot x_i^2$$


p² =
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
In practise all methods converge with increasing sample size
Bayes
Frequentist
Laplace
0.56
0.48
To bet or not to bet
Left
Left
Right
Left
Right
Left

Some more books about uncertainty and intuitive statistics

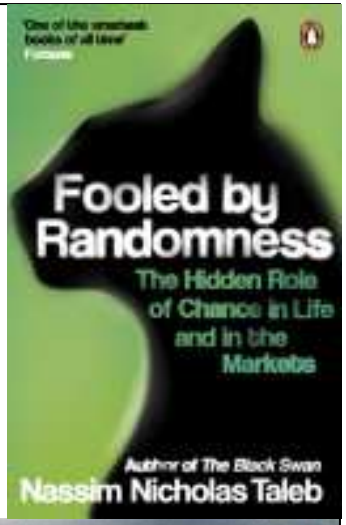
But above of all this "international best-seller"





Selected by the ***New York Times Book Review*** as one of the best books of 2011 A ***Globe and Mail*** Best "Books of the Year 2011". One of ***The Economist's*** 2011 "Books of the Year". One of ***The Wall Street Journal's*** "Best Nonfiction Books of the Year 2011"

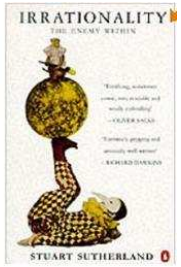
the theory that would not die
how bayes' rule cracked the enigma code, hunted down russian submarines & emerged triumphant from two centuries of controversy
sharon bertsch mcgrayne

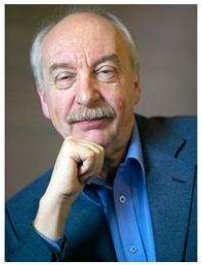


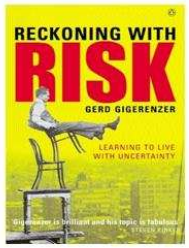
Hydrology 16 July 2013 4

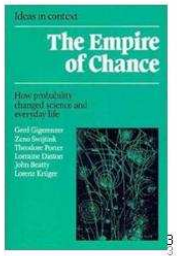
Good books on "intuitive statistics" and "rational thinking":

Stuart Sutherland (1994):
Irrationality – The Enemy Within

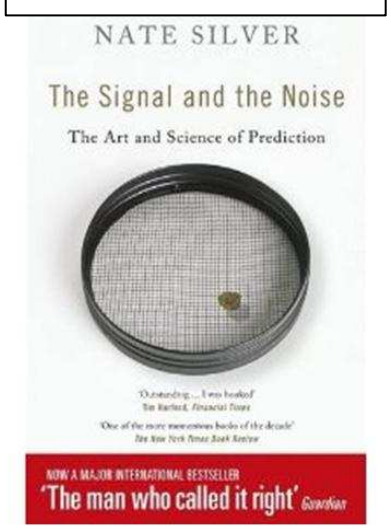


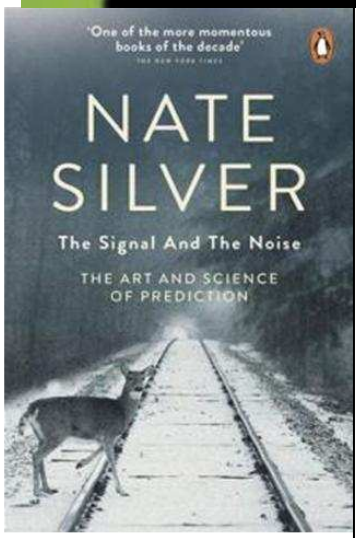






Gerd Gigerenzer on risk and chance





14/10/2013 Hydrology 16 July 2013 Anders Persson 3

Hydrology 16 July 2013 Anders Persson 5

END