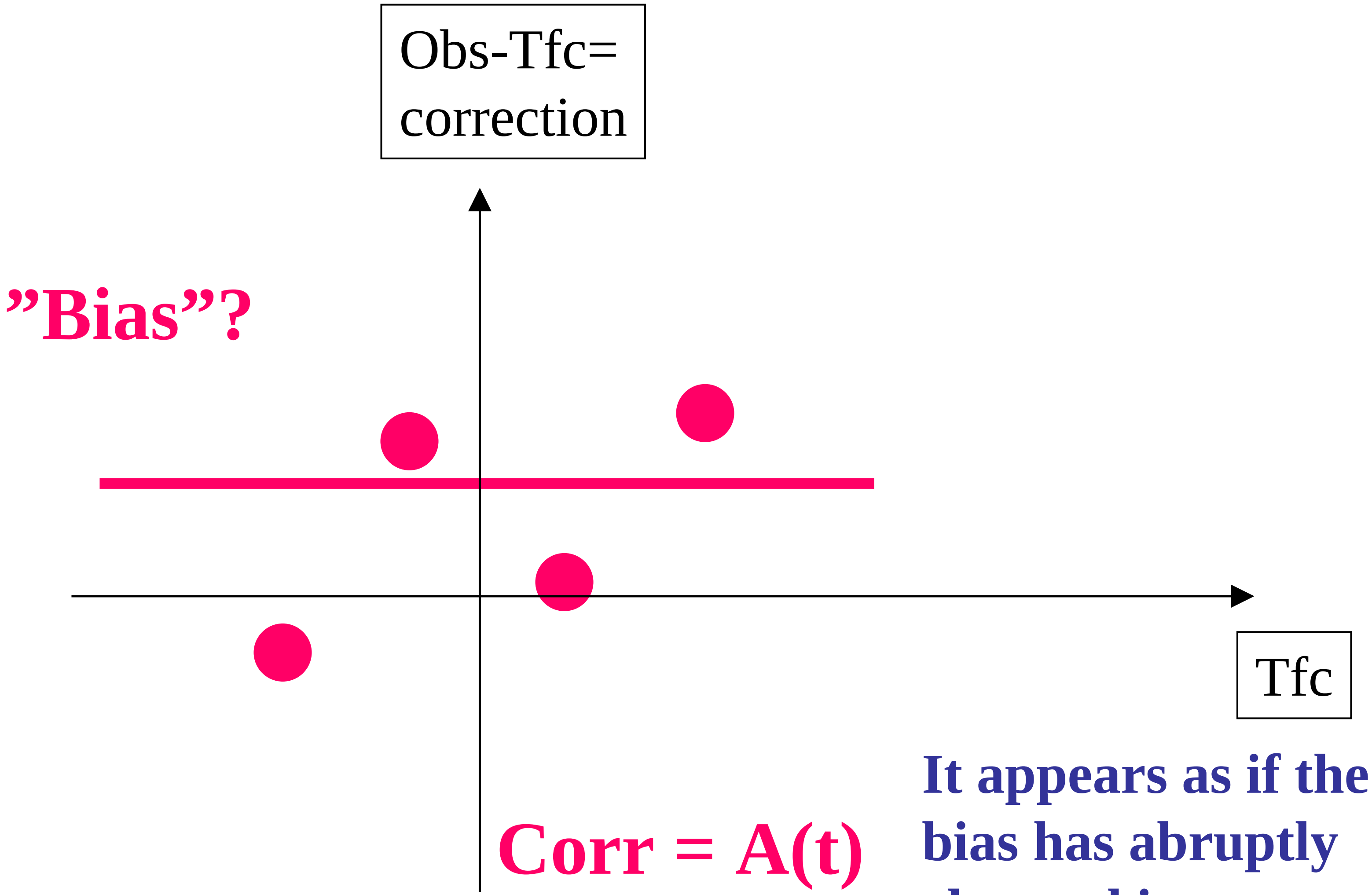
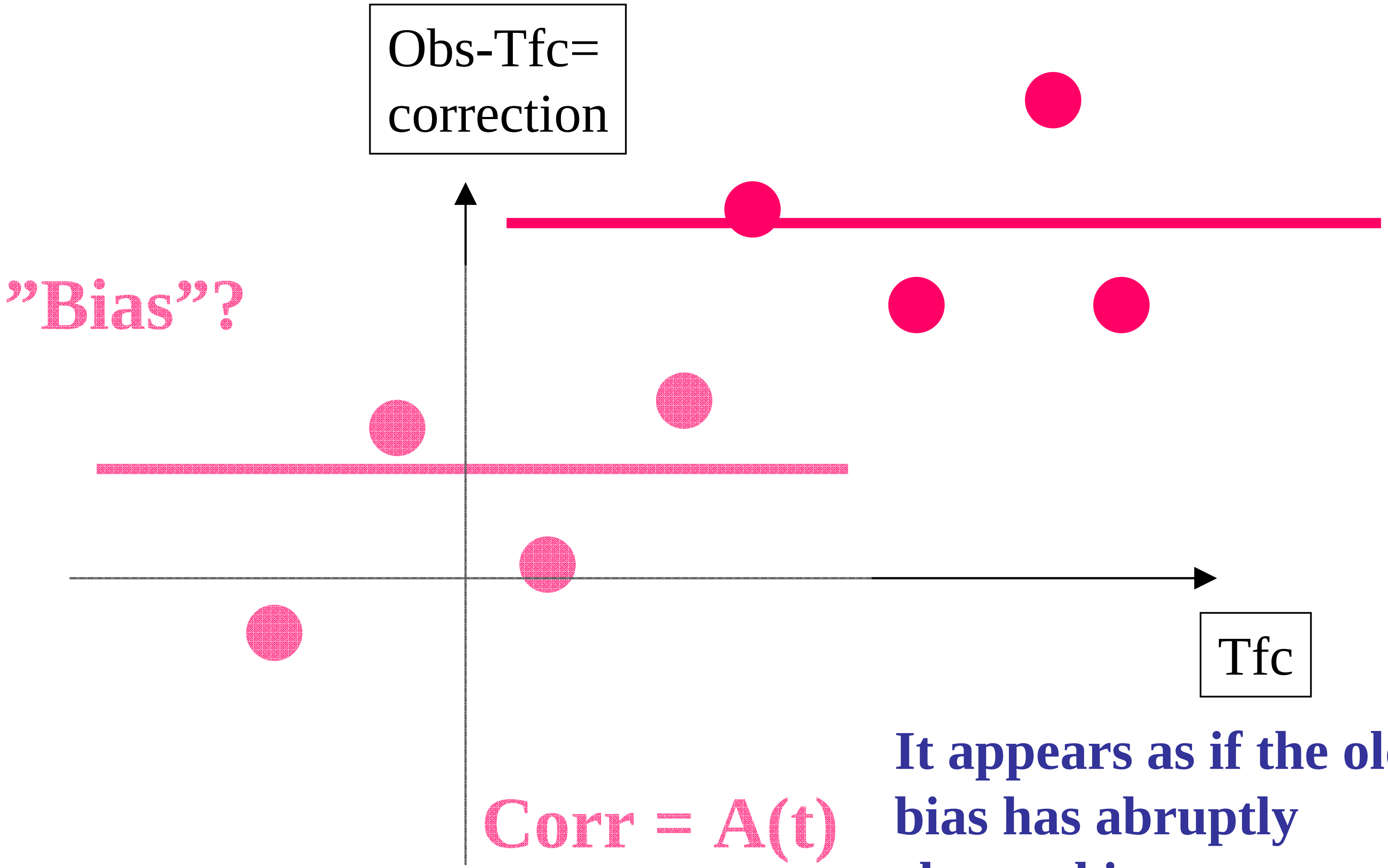
III Subjective probabilities

III.4. Adaptive Kalman filtering

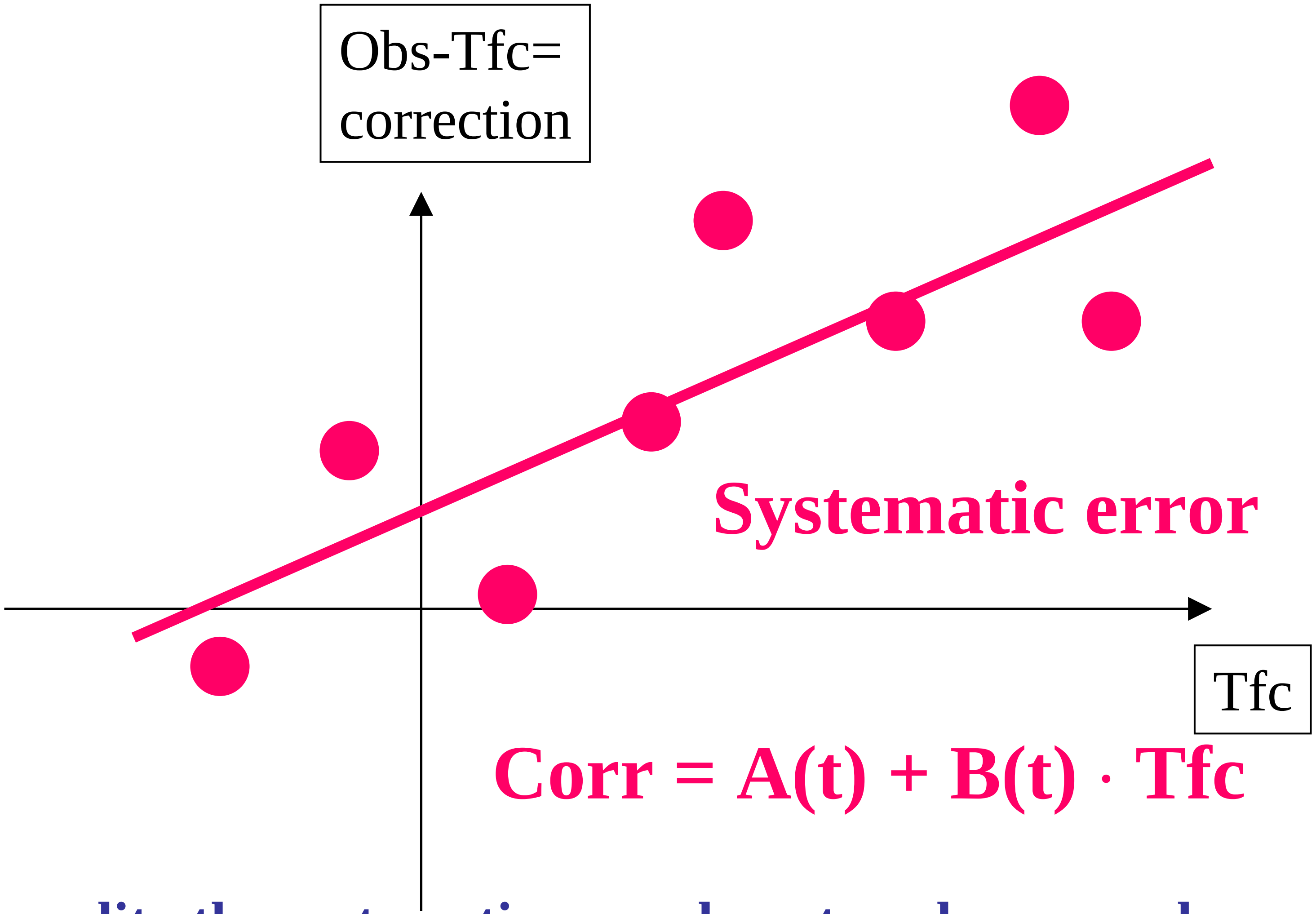
III.4.1 A 2-dimensional Kalman filter system



It appears as if the old bias has abruptly changed into a new one

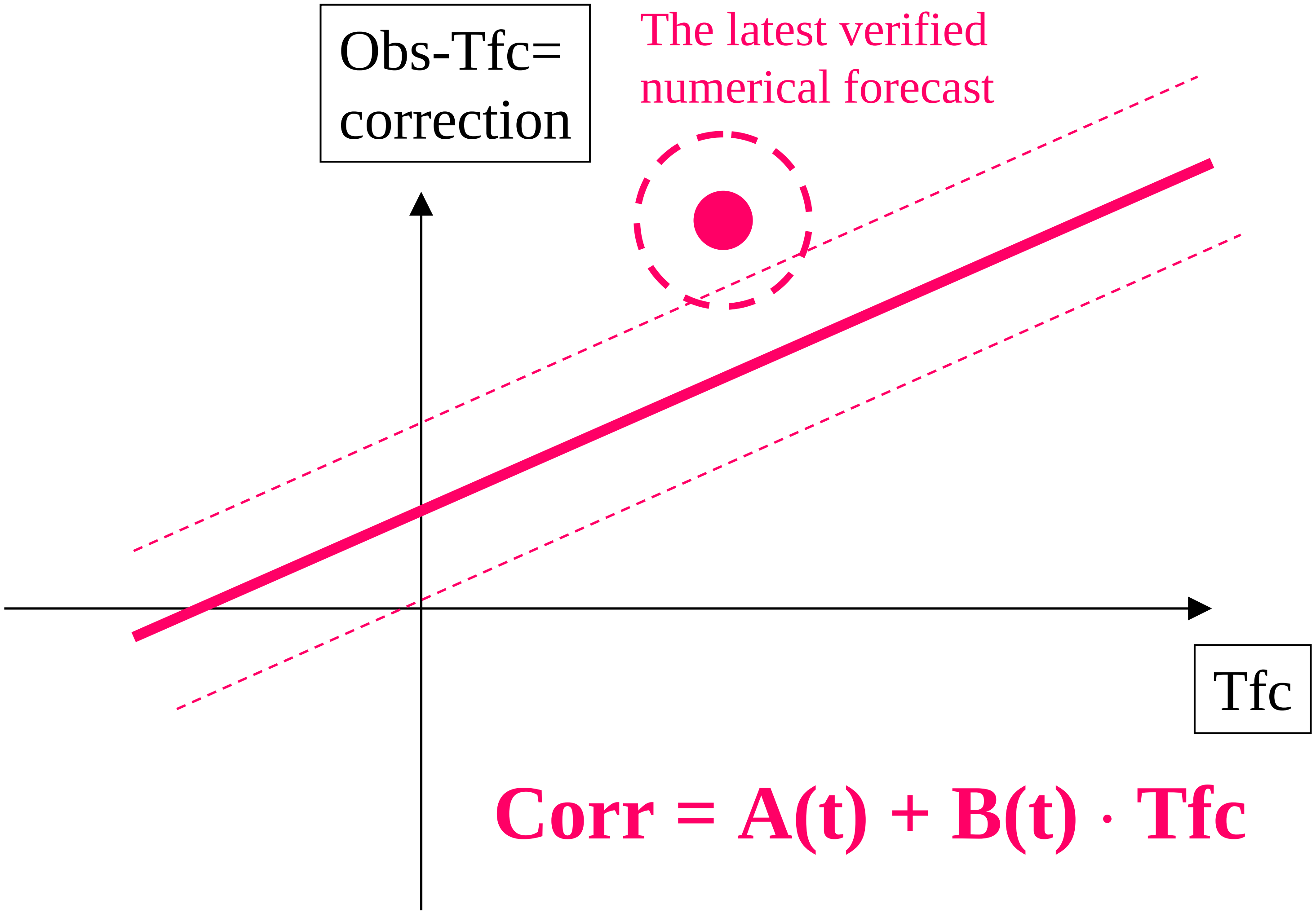


It appears as if the old bias has abruptly changed into a new one

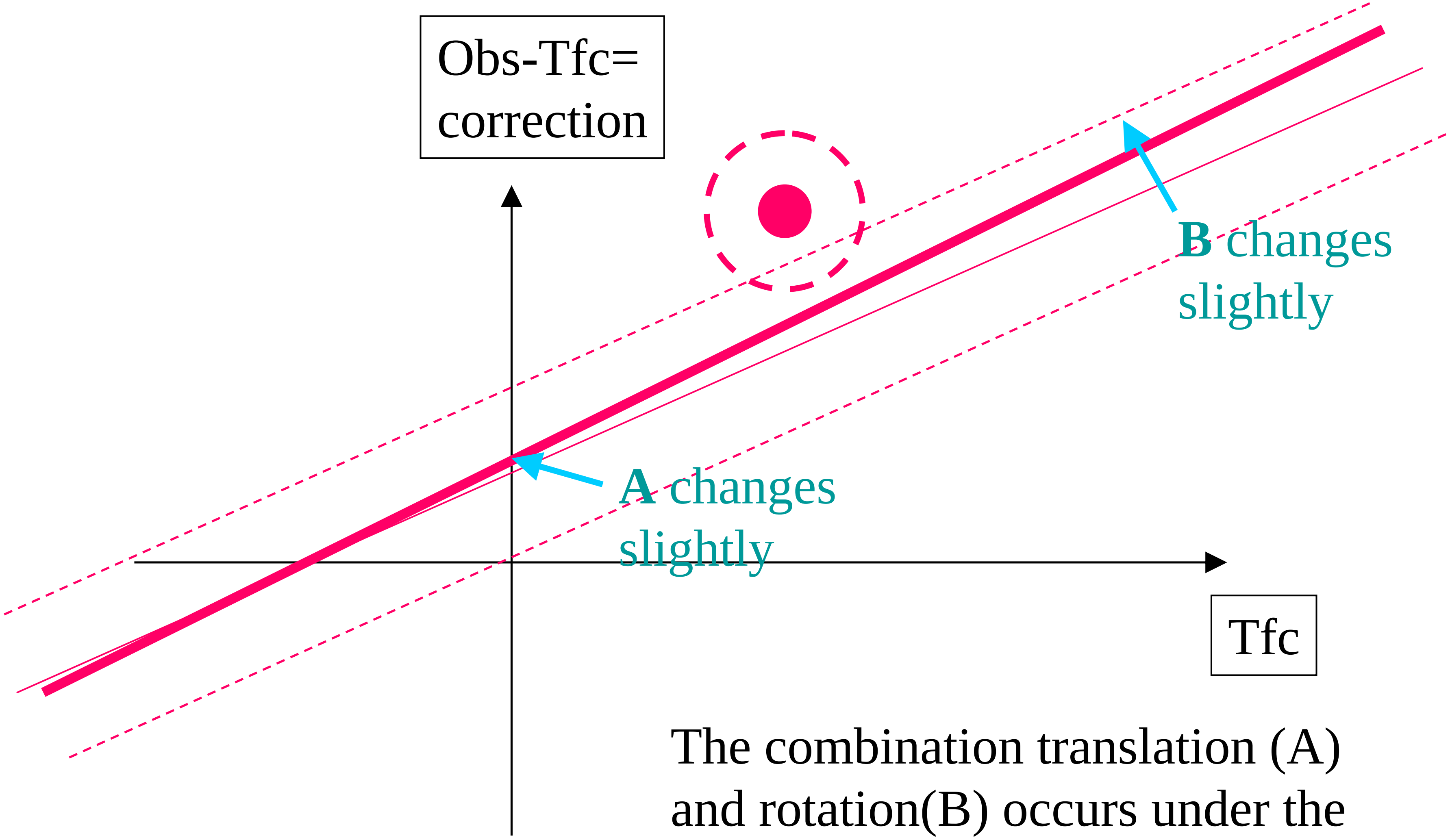


Corr = A(t) + B(t) · Tfc

In reality the systematic error has stayed more or less the same, but defined by *two* coefficients, A and B

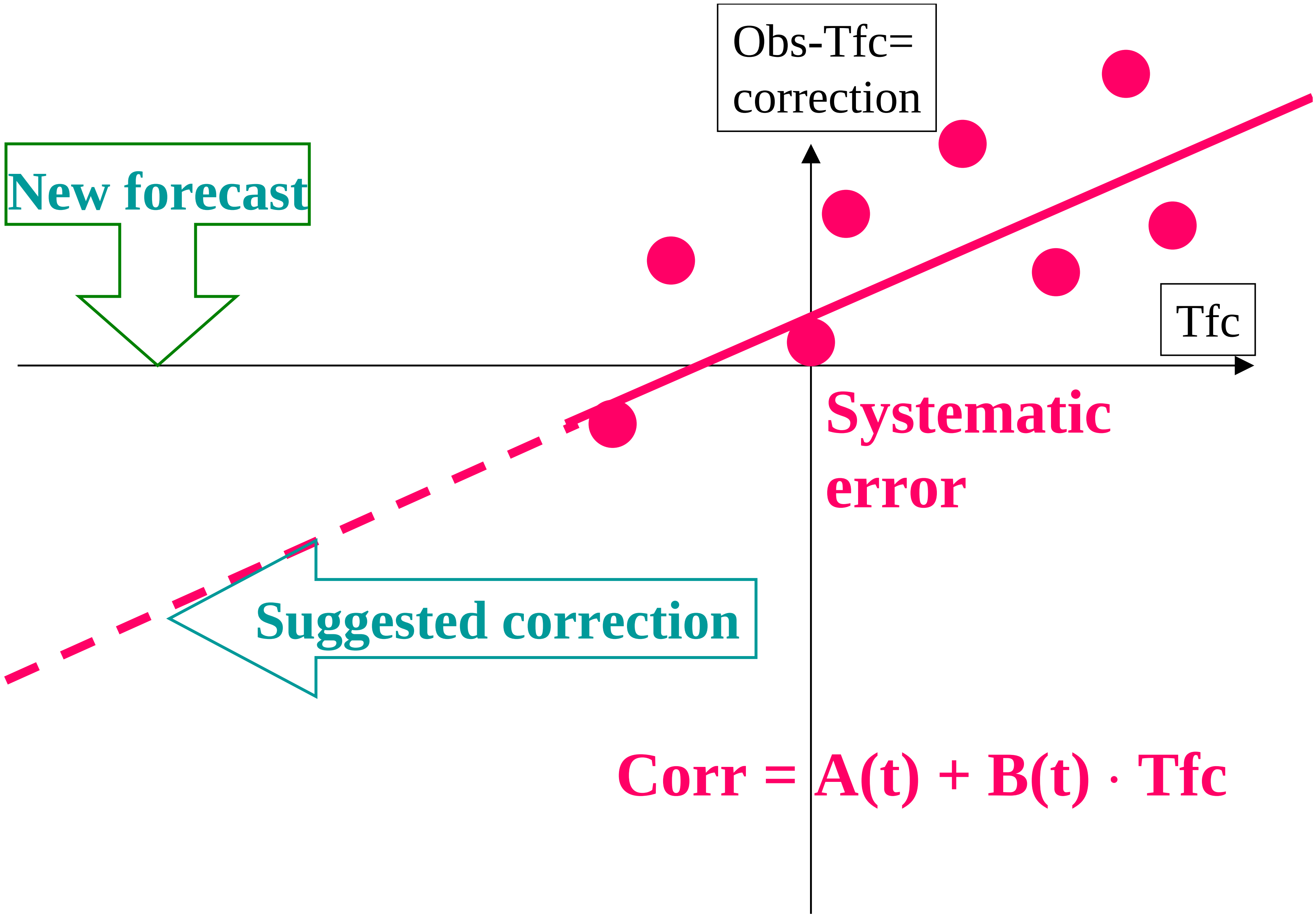


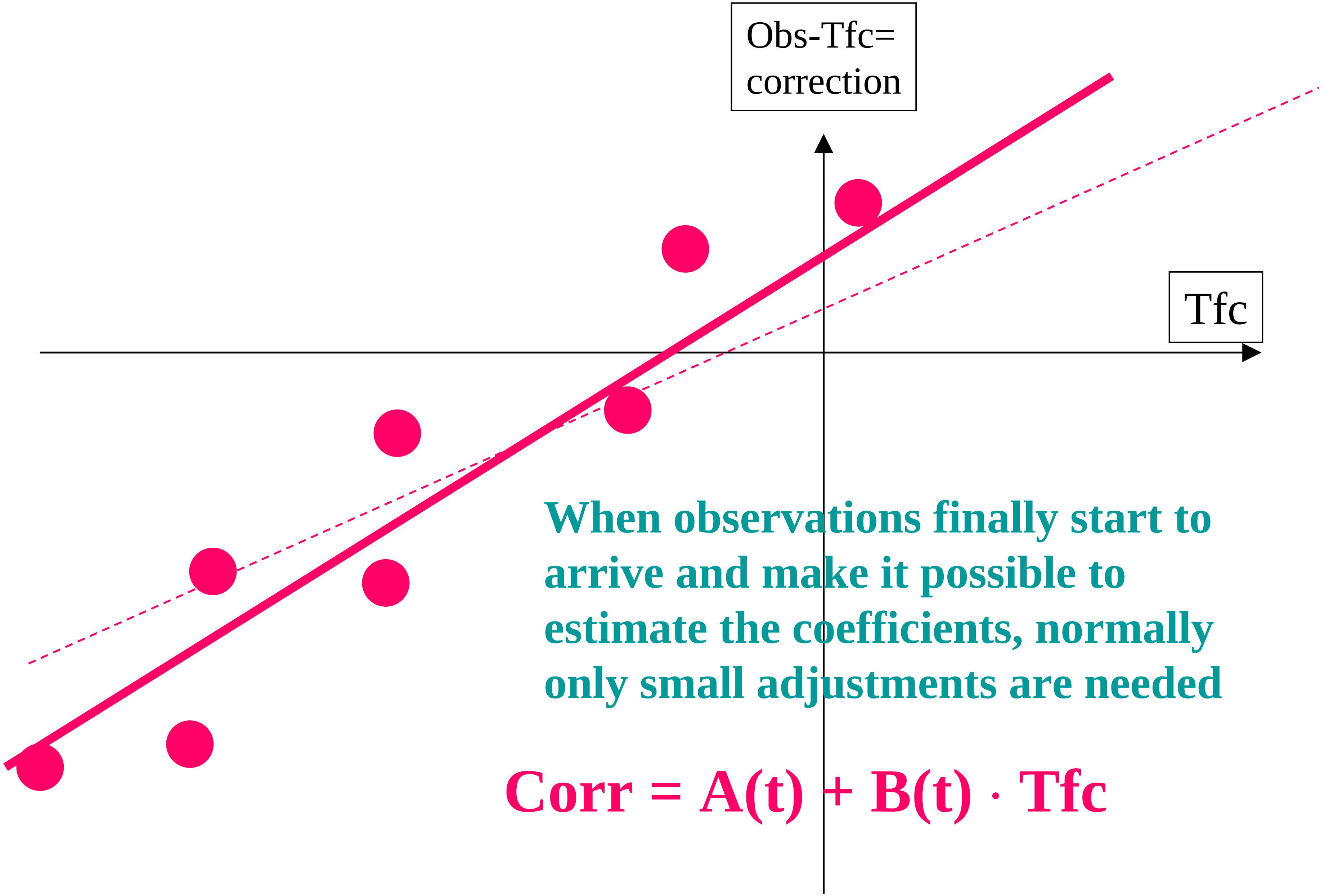
$$\mathbf{Corr = A(t) + B(t) \cdot Tfc}$$



The combination translation (A)
and rotation(B) occurs under the
variational condition of ”least effort”

Corr = A + B · Tfc

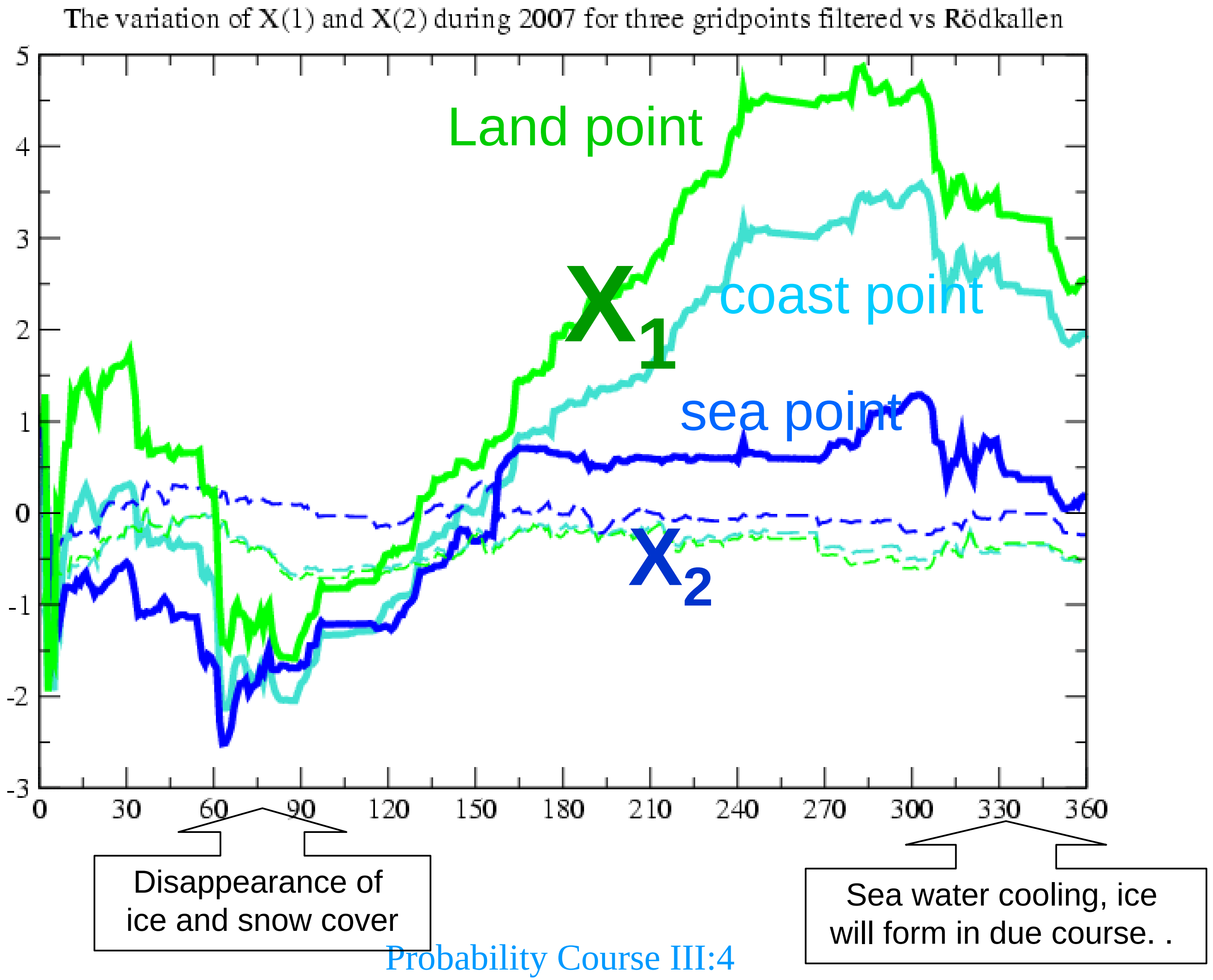

Obs-Tfc=
correction
New forecast
Tfc
Systematic
error
Suggested correction
Corr = A(t) + B(t) · Tfc



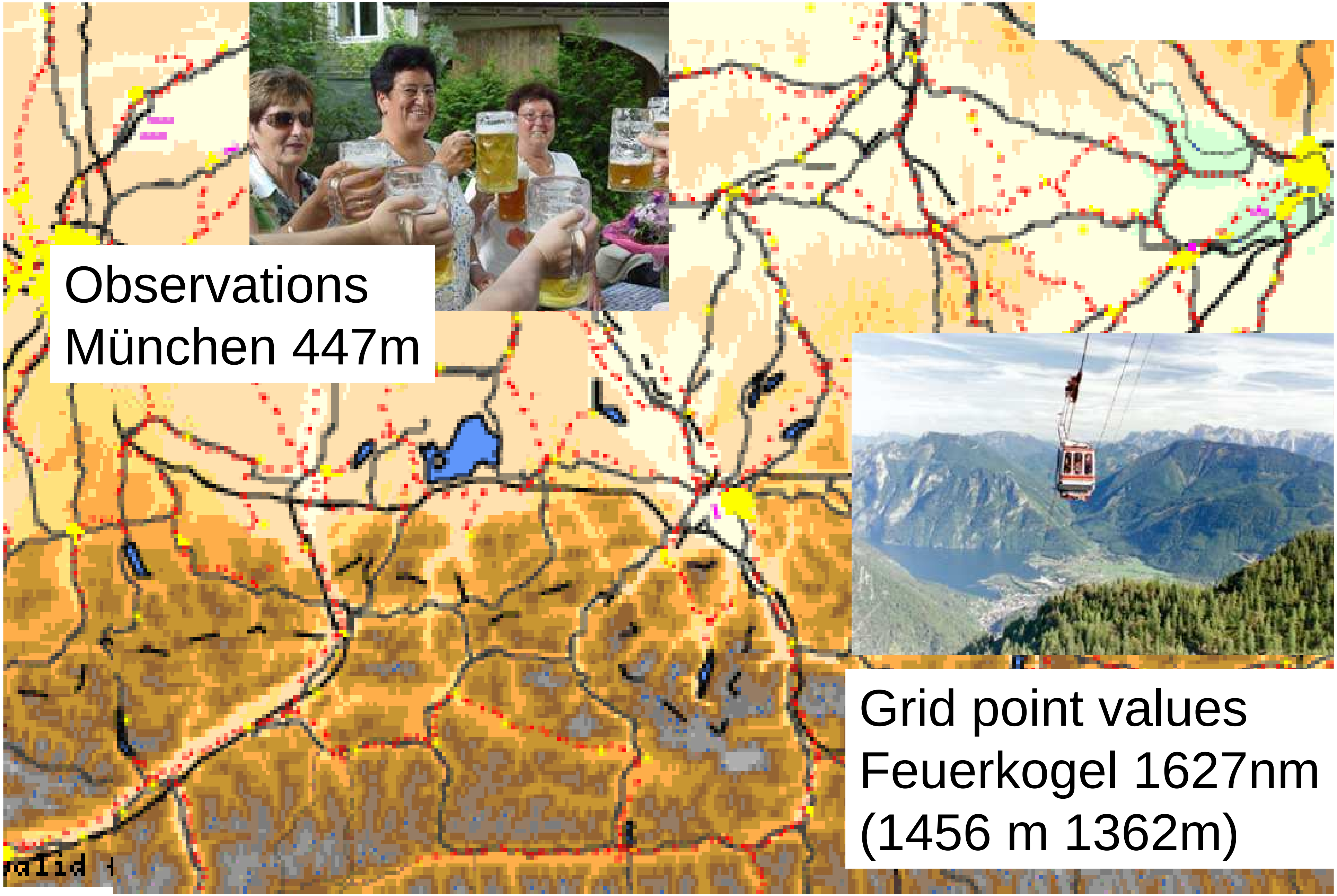
When observations finally start to arrive and make it possible to estimate the coefficients, normally only small adjustments are needed

$$\mathbf{Corr = A(t) + B(t) \cdot Tfc}$$

The variation of the coefficients indicate significant changes in model and/or environment

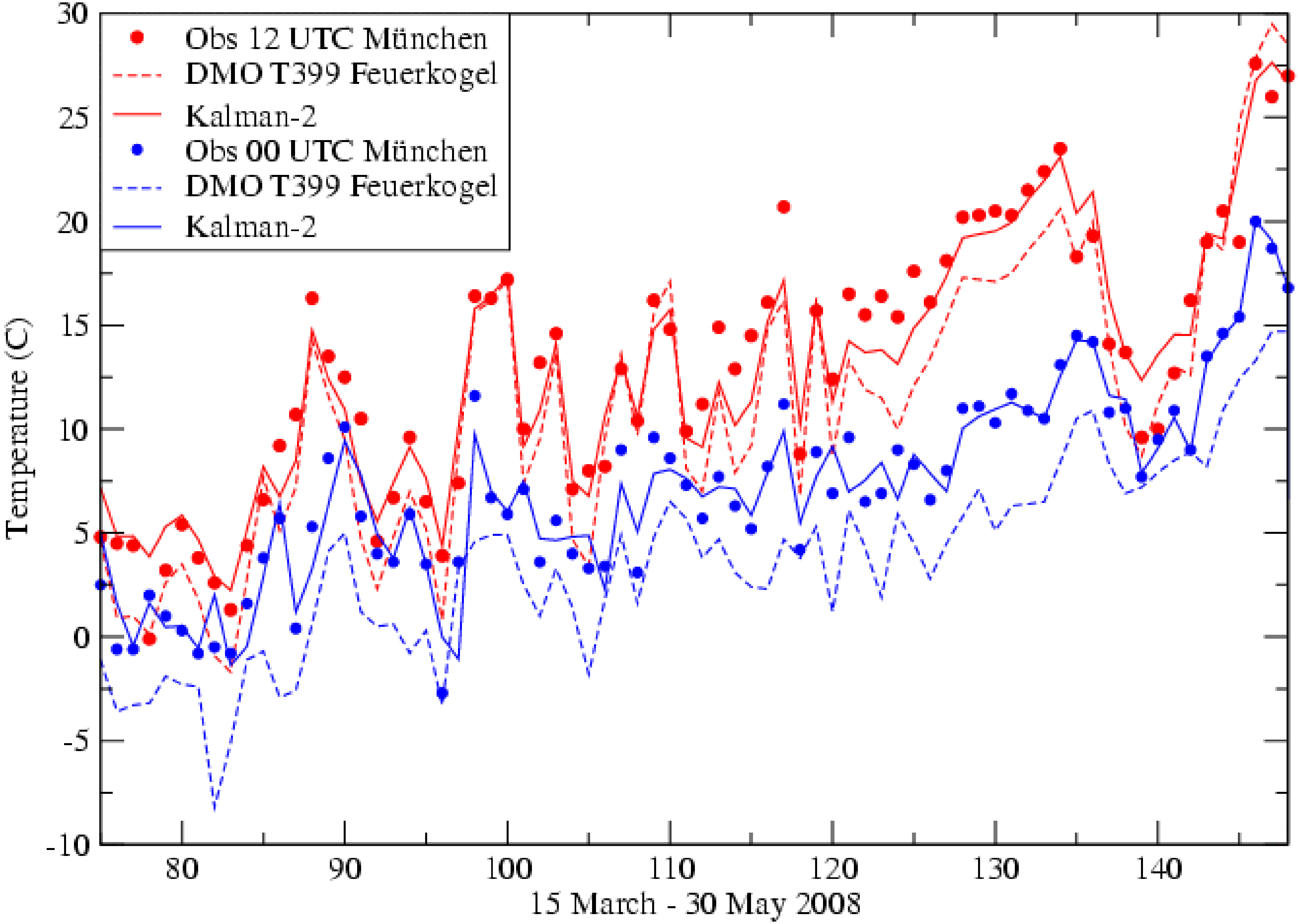


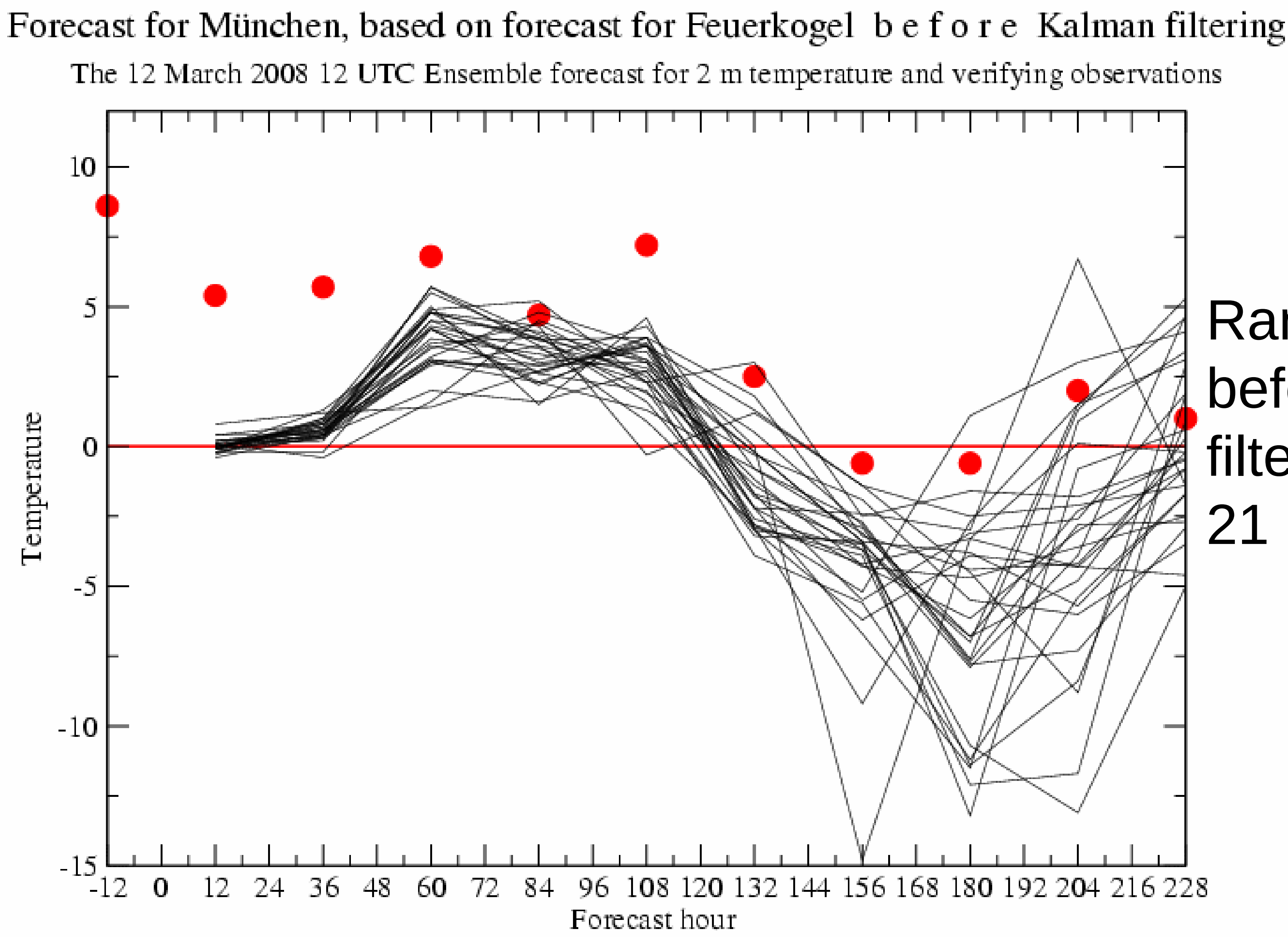
III.4.2 Station and grid point can be far away!


Observations
München 447m
Grid point values
Feuerkogel 1627nm
(1456 m 1362m)

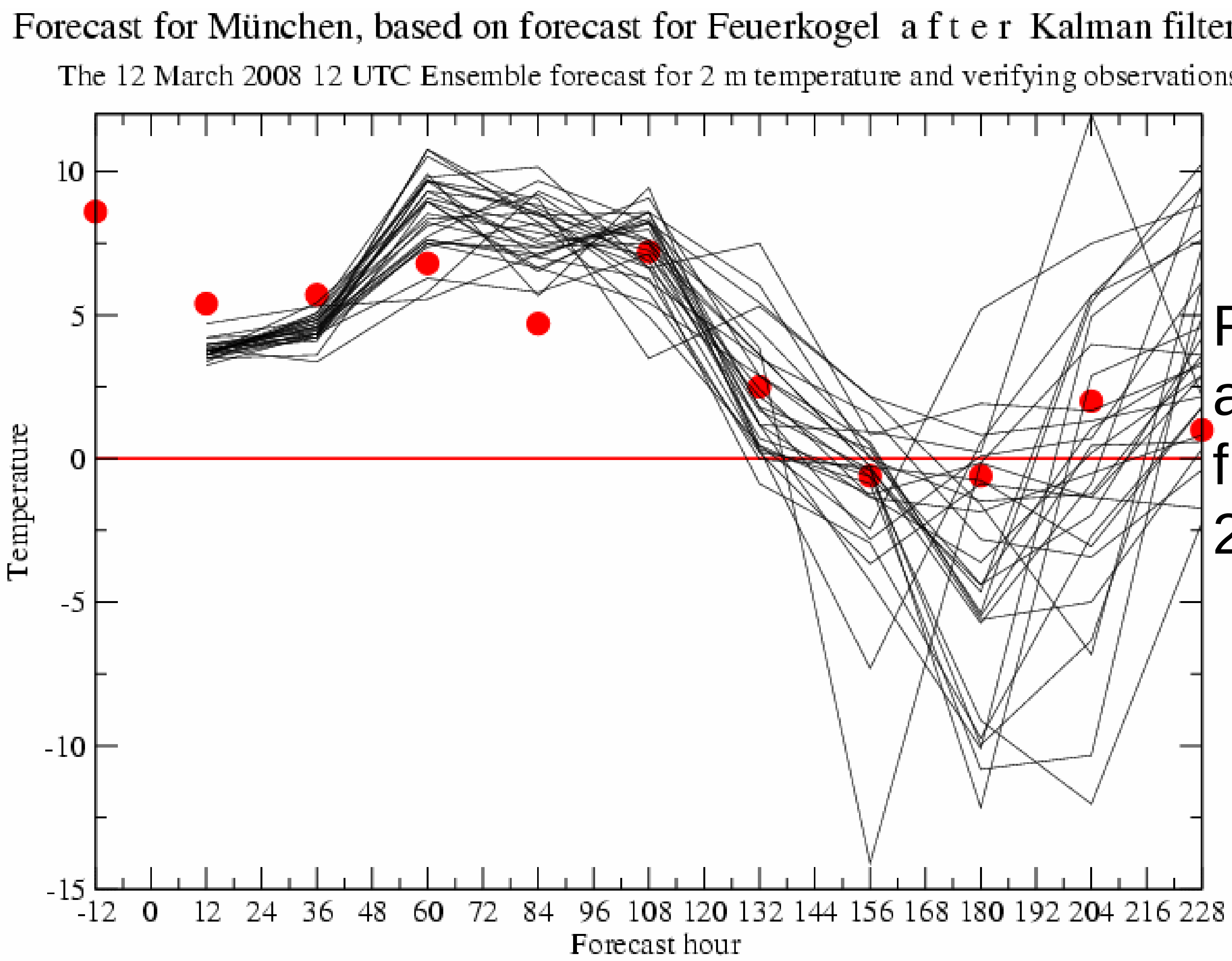
Kalmanfiltering ECMWF D+1 forecasts for München (447 m)

...against ECMWF D+1 EPS Control forecasts for Feuerkogeln 1362 m (model height)

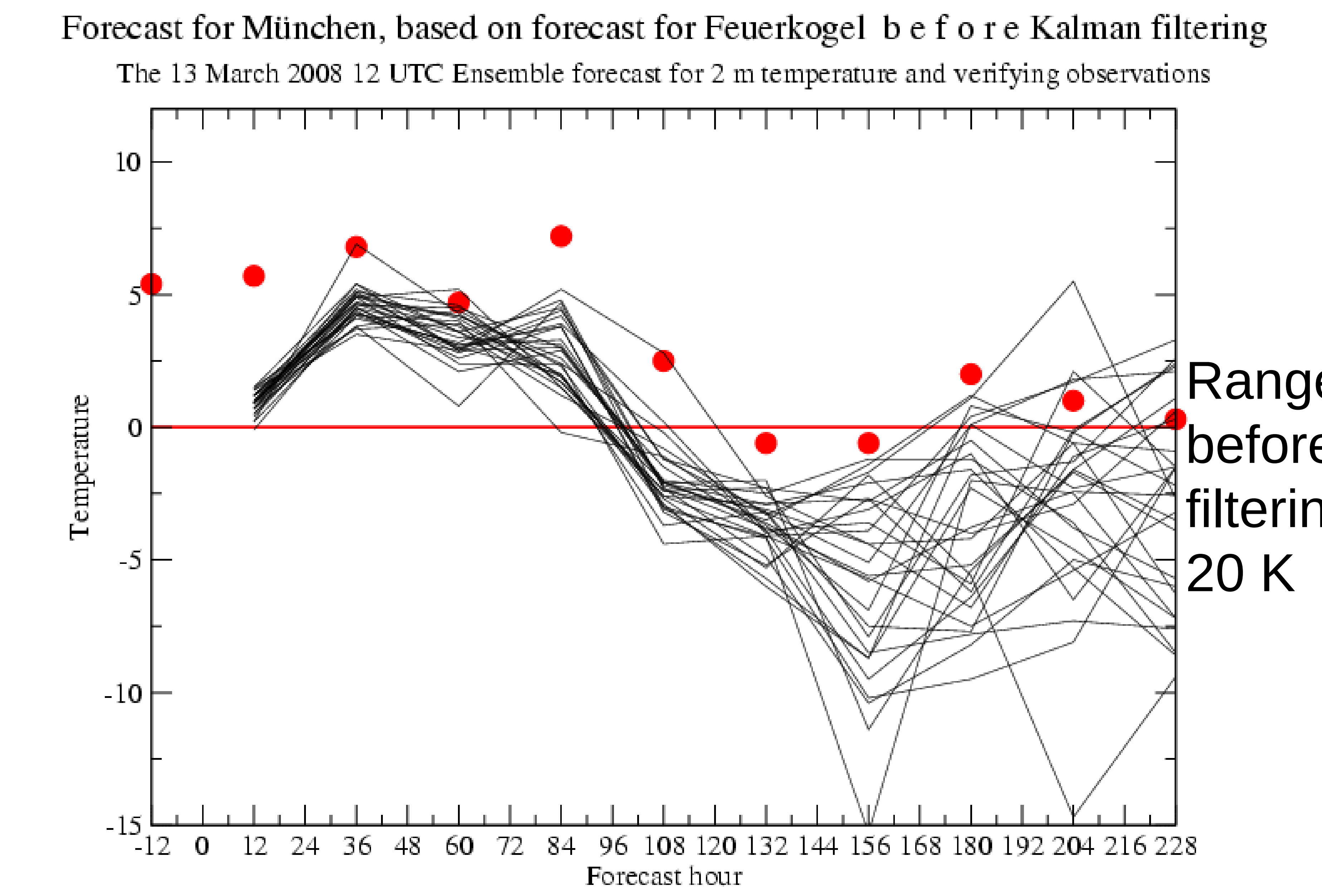




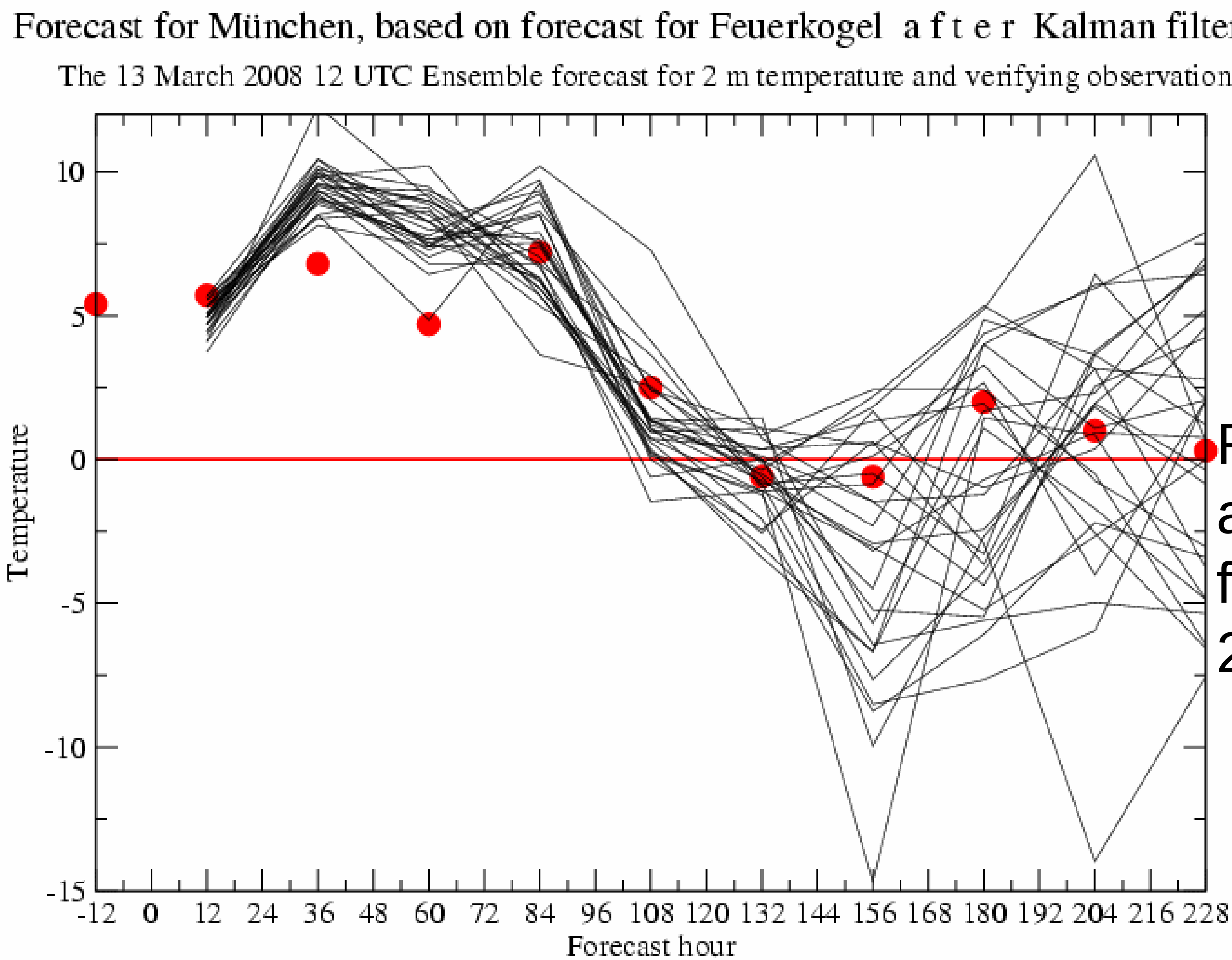
Range before filtering 21 K



Range after filtering 27 K



Range before filtering 20 K

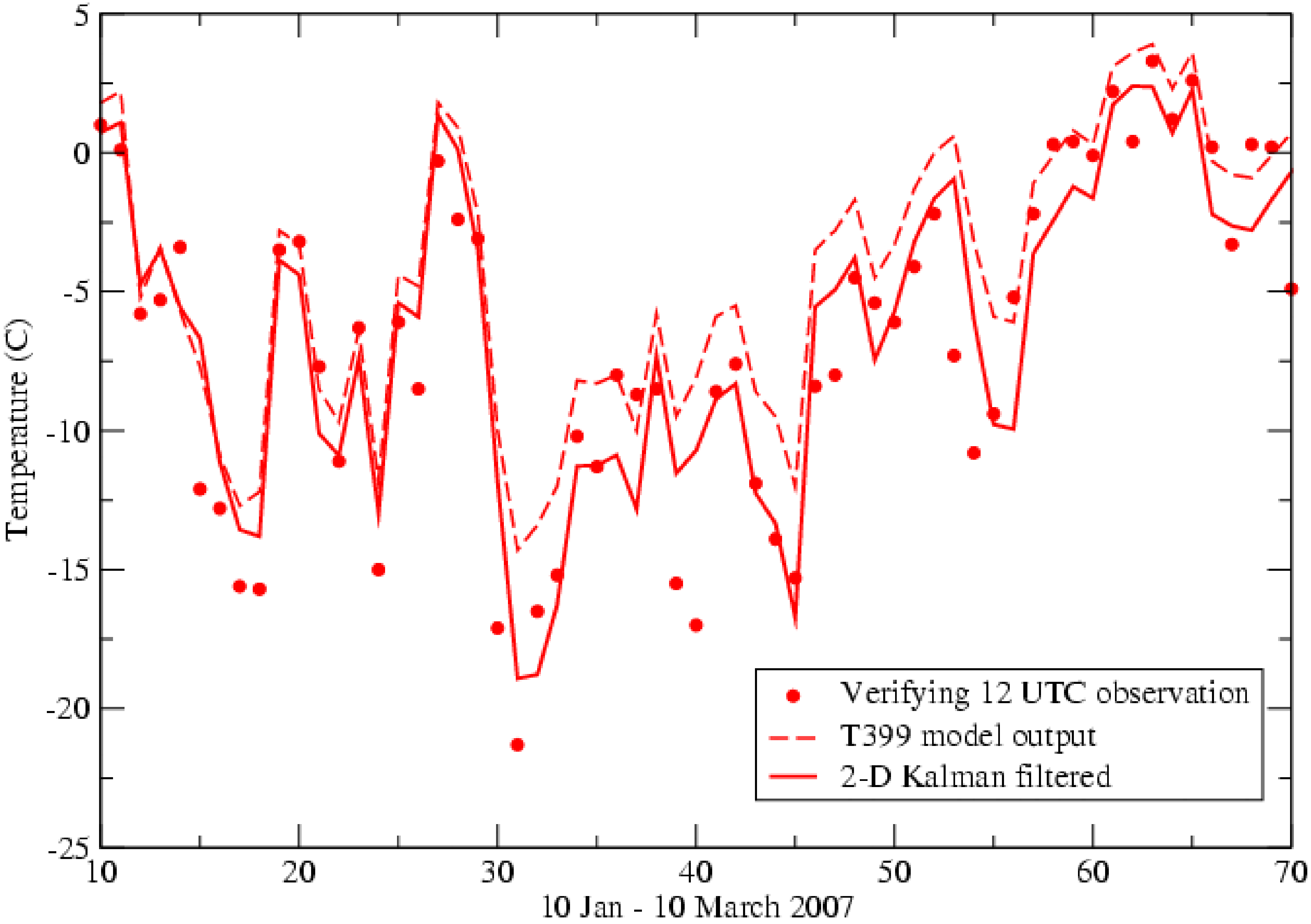


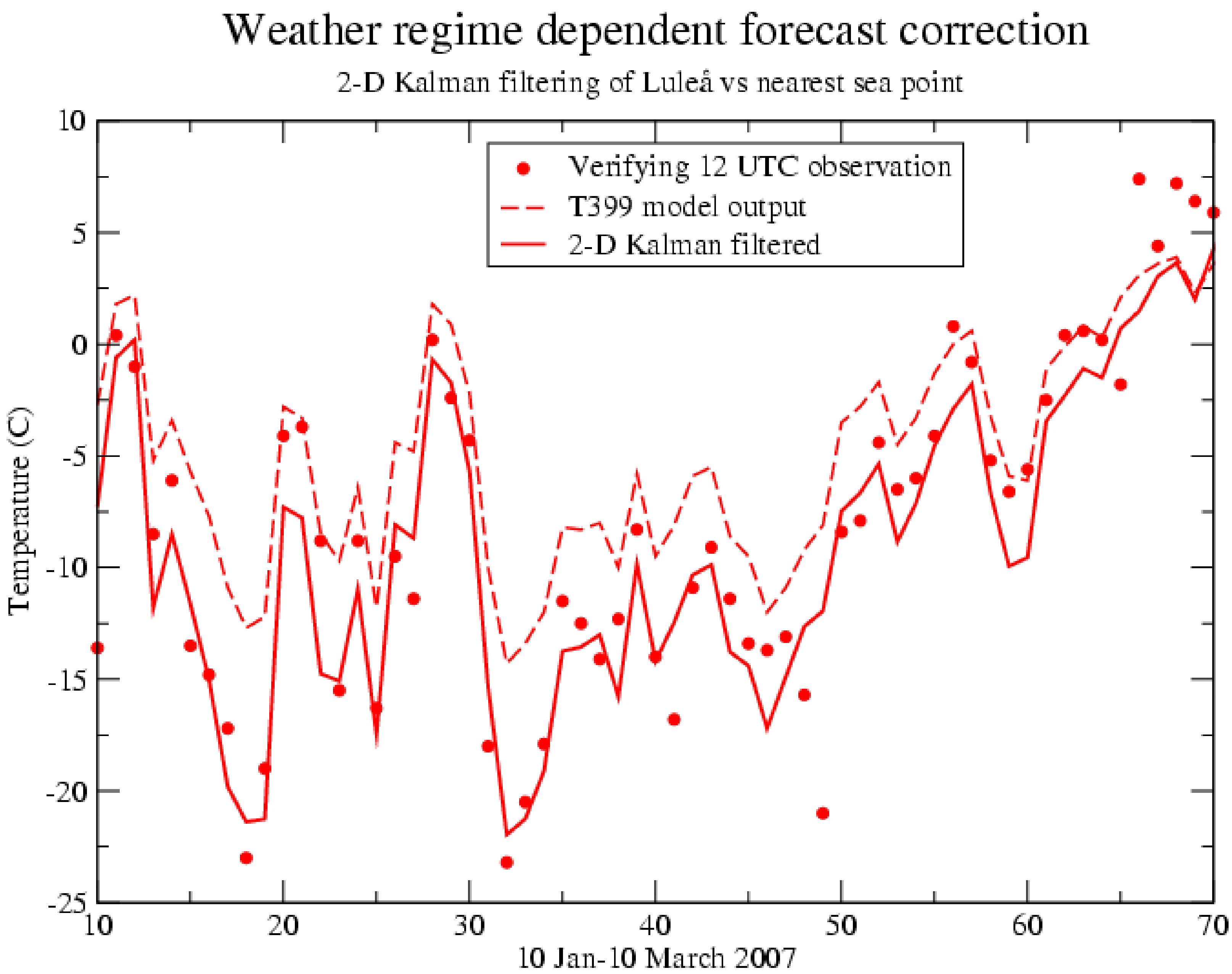
Range after filtering 26 K

III.4.3. The 2- or N-dimensional filter does not only correct mean errors (”biases”) but also systematic over- and under variability

Weather regime dependent forecast correction

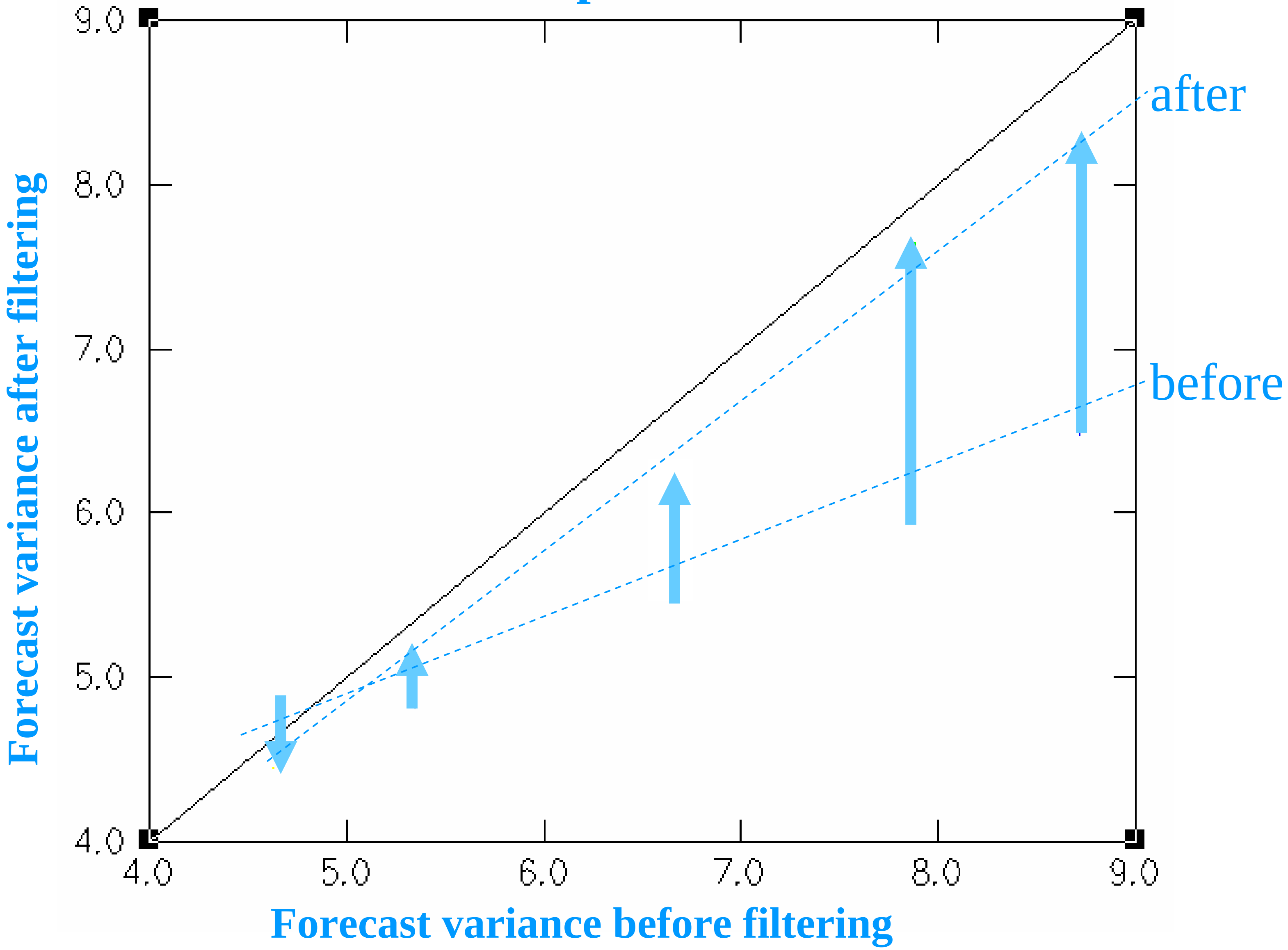
2-D Kalman filtering of Rödkallen lighthouse vs nearest sea point

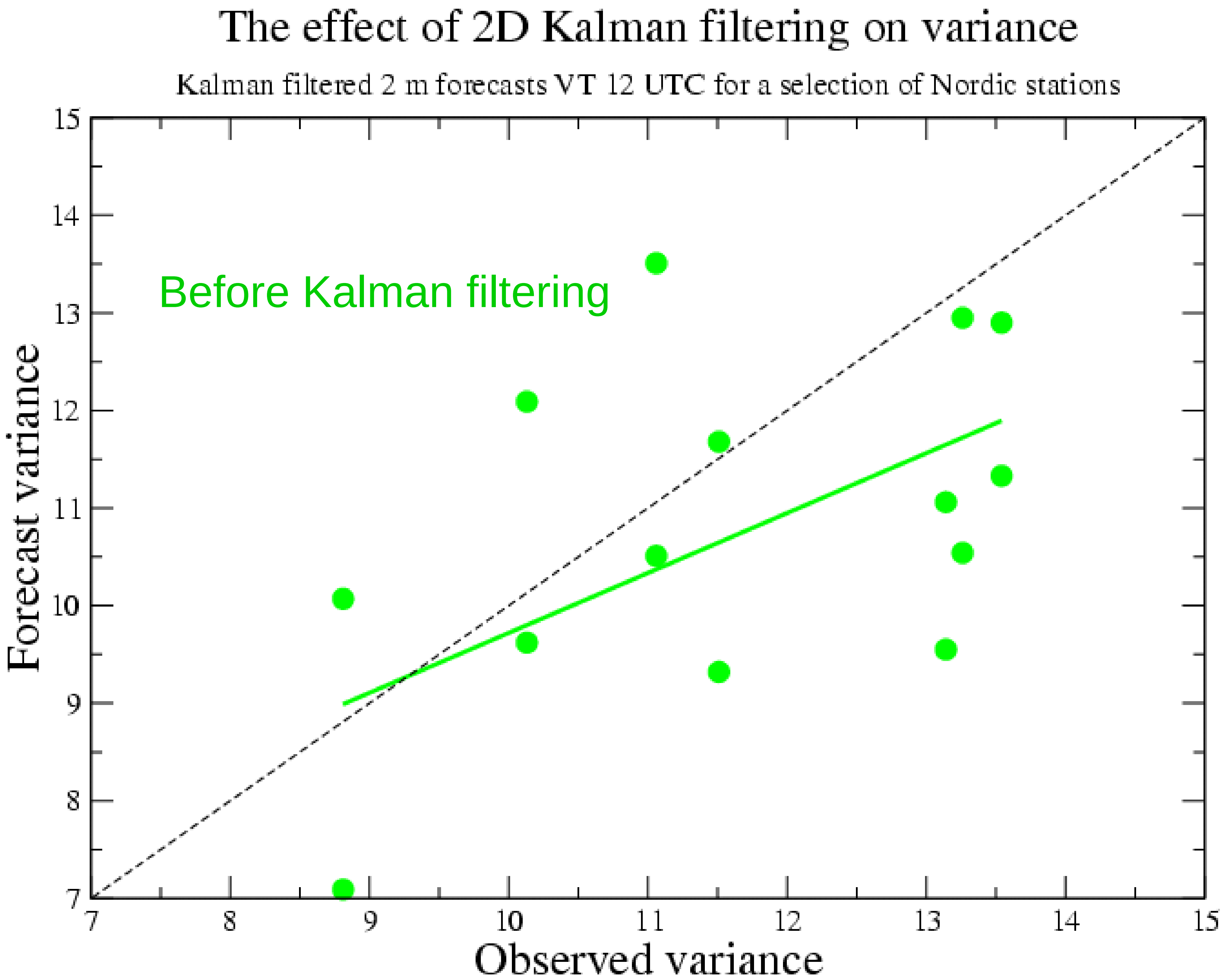




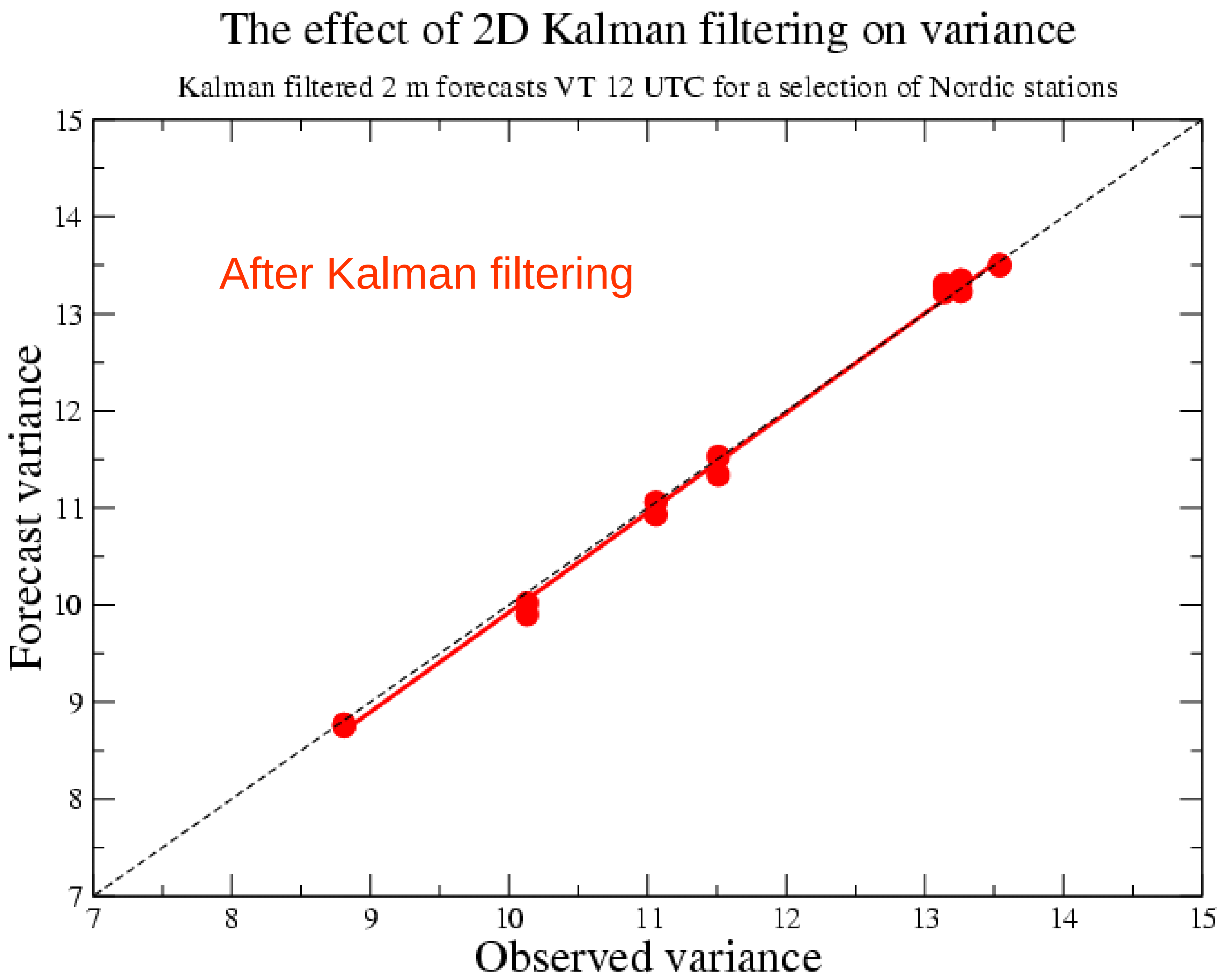
Weather regime dependent forecast correction
2-D Kalman filtering of Luleå vs nearest sea point
Verifying 12 UTC observation
T399 model output
2-D Kalman filtered
Temperature (C)
10 Jan-10 March 2007
10
5
0
-5
-10
-15
-20
-25
10
20
30
40
50
60
70

The Kalman 2 filter improvement of forecast variance





The effect of 2D Kalman filtering on variance
Kalman filtered 2 m forecasts VT 12 UTC for a selection of Nordic stations
Before Kalman filtering
Forecast variance
Observed variance
7
8
9
10
11
12
13
14
15


The effect of 2D Kalman filtering on variance
Kalman filtered 2 m forecasts VT 12 UTC for a selection of Nordic stations
After Kalman filtering
Forecast variance
Observed variance
7
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12
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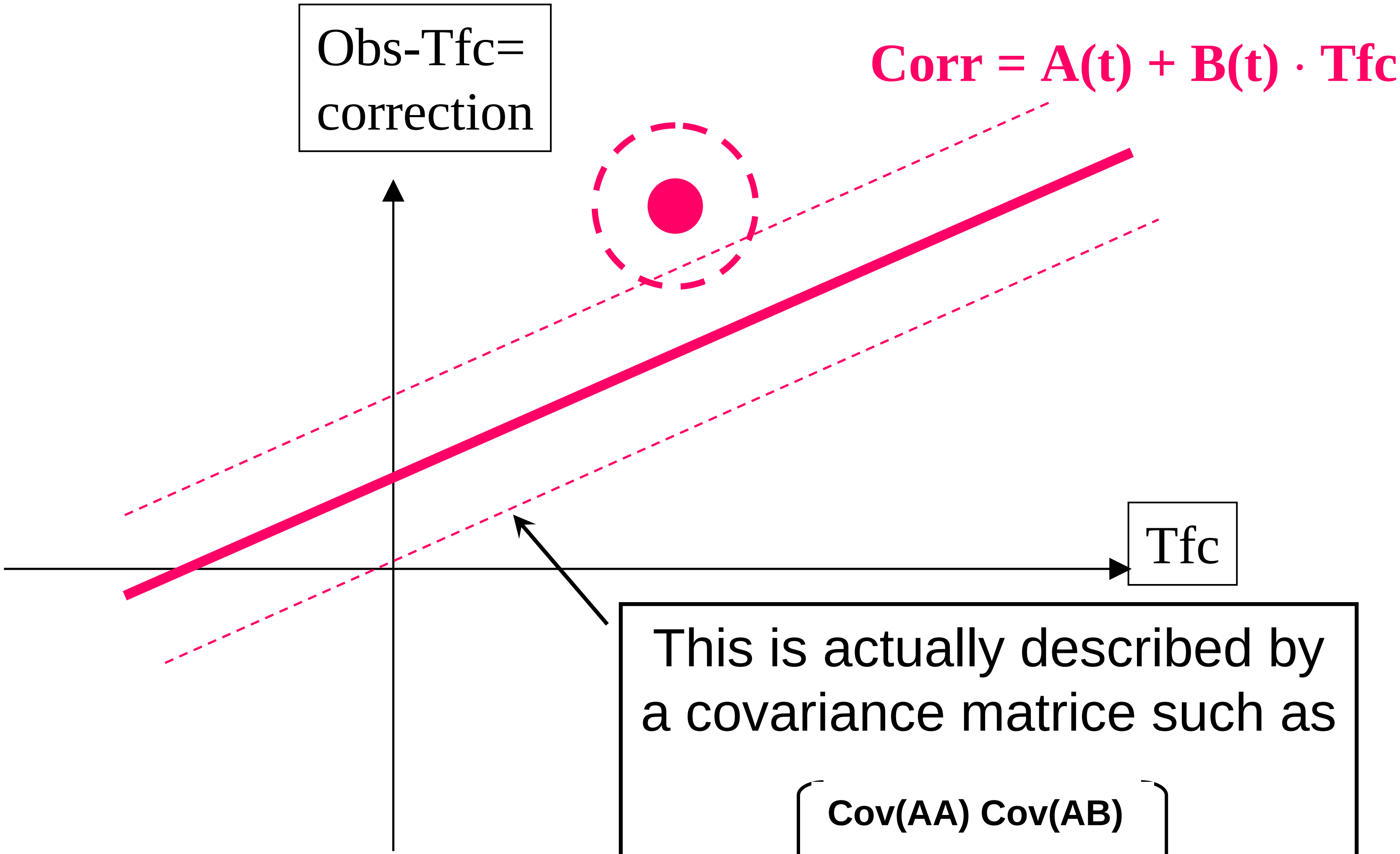
14
15

III.4.4 Further improvement of the spread



This is actually described by a covariance matrice such as

$$\begin{pmatrix} \mathbf{Cov(AA)} & \mathbf{Cov(AB)} \\ \mathbf{Cov(BA)} & \mathbf{Cov(BB)} \end{pmatrix}$$

The ECMSWF Kalman filters

The covariance matrices in Kalman filter used in “Ensemble Kalman Filtering” only has non-zero values in the diagonals

cov(A,A)	**0**
0	**cov(B,B)**

But that is because its covariance matrices are in dimension not of 2, 3 or 4 but in 10^6

Expected error $dT = A_t + B_t \cdot Fc$

The Kalman filter will now provide a 2-dim variance matrix

$$\mathrm{Cov}(\mathbf{AB}) = \begin{pmatrix} \mathrm{cov}(A,A) & \mathrm{cov}(A,B) \\ \mathrm{cov}(B,A) & \mathrm{cov}(B,B) \end{pmatrix}$$

$$\mathrm{Variance}(\mathbf{dT}) = E\{\mathbf{dT}^2\} =$$

$$E\{(\mathbf{A} + \mathbf{B} \cdot \mathbf{F}_C)^2\} = E\{\mathbf{A}^2\} + E\{\mathbf{B}^2\} \cdot \mathbf{F}_C^{\,2} + 2E\{\mathbf{AB}\} \cdot \mathbf{F}_C$$

yields $\mathbf{Var(A) + F_C^{\,2}\,Var(B) + 2F_C Cov(AB)}$

A practical example:

The 2-dim Kalman filter system has found that the error equation

$$\text{Expected error } dT = 0.7 + 0.2 \cdot F_c$$

provides the best estimation, which for F_c=5.0° yields a correction of dT = 1.7°. Assume the covariance matrix

$$\begin{pmatrix} 0.0400 & 0.0100 \\ 0.0100 & 0.0025 \end{pmatrix}$$

cov(A)

F_c^2cov(B)

$2F_c$cov(AB)

$$\text{Var}(dT) = 0.0400 + 25 \cdot 0.0025 + 2 \cdot 5 \cdot 0.0100 = 0.2025$$

or a standard deviation dT = 0.45°

which, as representing small scale uncertainty, in an ensemble application, should be added to the large scale, synoptic-dynamic uncertainty.

III.4.5 The Joseph Form

The covariance update equation:

"My" update of coefficient and covariances

$$\mathbf{P}_{t/t} = (\mathbf{I} - \mathbf{k}_t\mathbf{f}_t)\mathbf{P}_{t/t-1}(\mathbf{I} - \mathbf{k}_t\mathbf{f}_t)^T + \mathbf{k}_t r_t \mathbf{k}_t^T$$

But according to most textbooks:

$$\mathbf{P}_{t/t} = \mathbf{P}_{t/t-1}(\mathbf{I} - \mathbf{k}_t\mathbf{f}_t)^T$$

Dipartimento di Elettronica e Informazione
POLITECNICO DI MILANO

Numerical methods in identification, control and signal processing: Kalman filtering

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Probability Course III:4
Bologna 9-13 February 2015

Symmetry of P: the Joseph form

The covariance propagation equations are given by

$$P_k(-) = \Phi_{k-1} P_{k-1}(+) \Phi_{k-1}^T + Q_{k-1}$$

$$P_k(+) = \left[I - \bar{K}_k H_k\right] P_k(-)$$

The first equation already guarantees symmetry.
The second can be equivalently written as

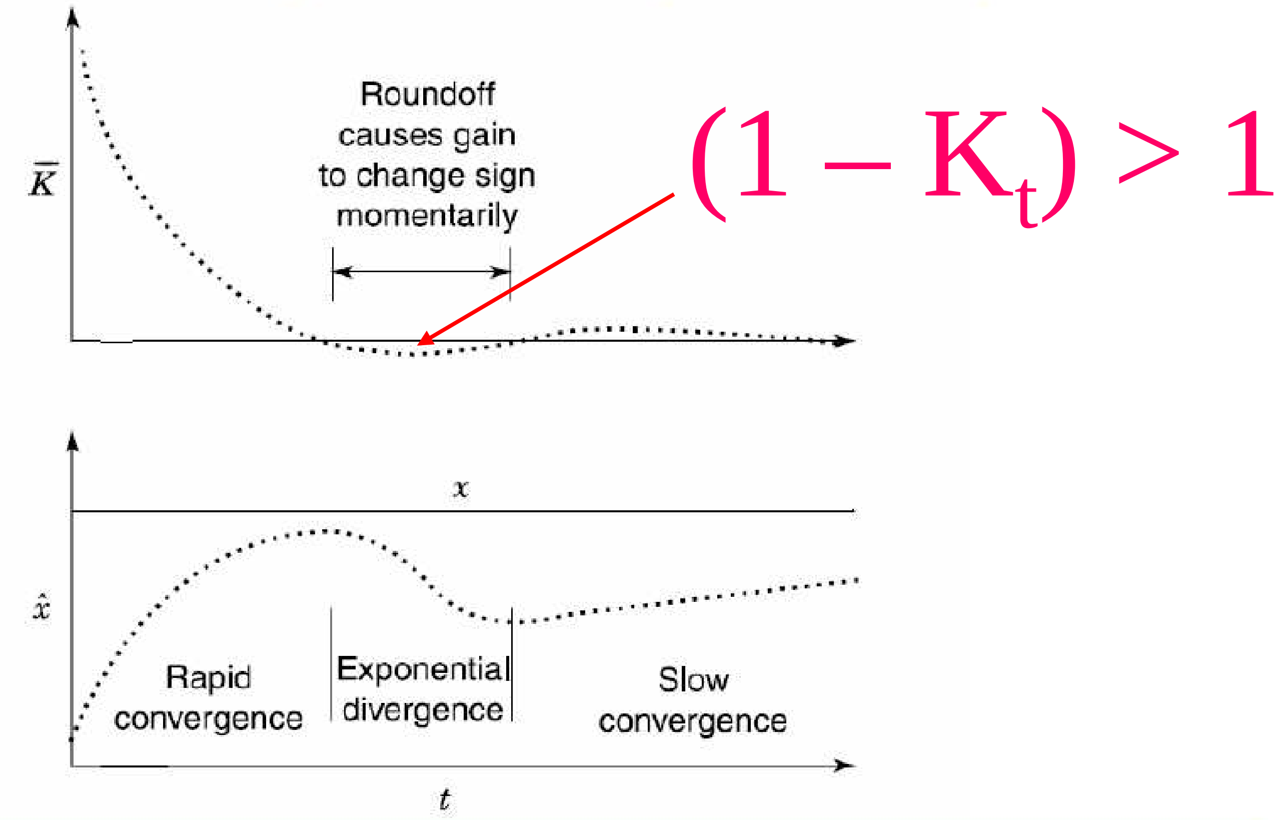
$$P_k(+) = \left[I - \bar{K}_k H_k\right] P_k(-) \left[I - \bar{K}_k H_k\right]^T + \bar{K}_k R_k \bar{K}_k^T$$

which again guarantees symmetry.

NOTE: this is the least one can do in implementing a KF!

Propagation of roundoff errors in KFs (2)

Example: what happens if the sign of P changes?



END