I. Classical probabilities

I.2 The power of randomness

I.2.1. What is “probability”?

Andrei Kolmogorov's definition of probabilities, 1933

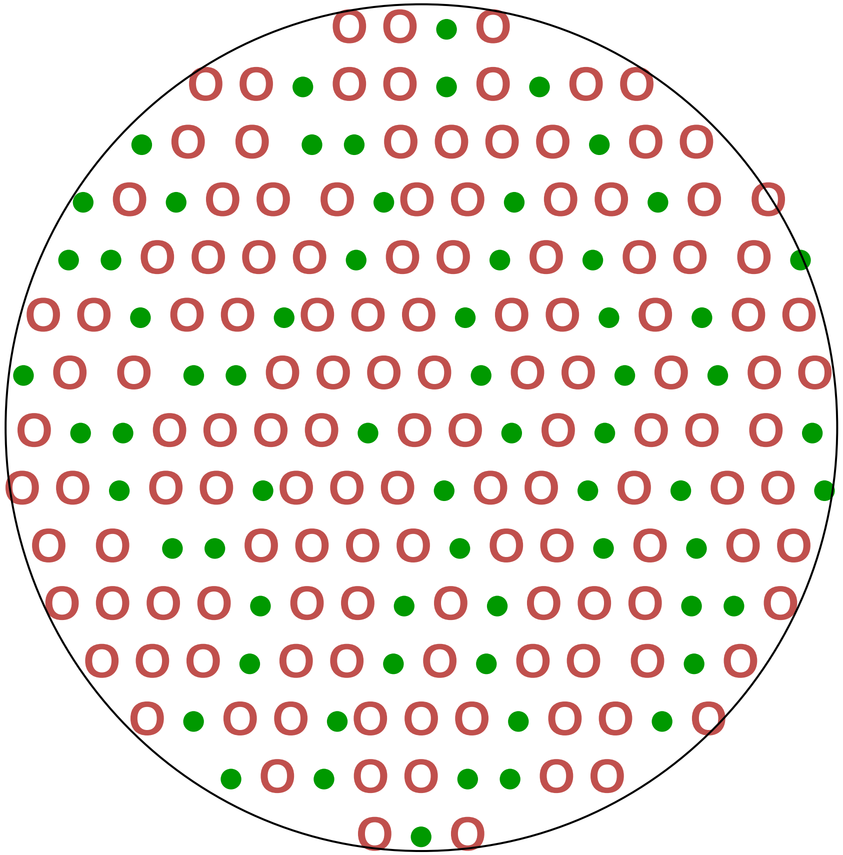
1. **Probability for any event = 100%**
2. **Probability for one type of events = F/N**
3. **Probability for several mutually exclusive events = (F+G+H)/N**

There are three types of probabilities: **the classical, the frequentist and the Bayesian**

1. **<u>The classical</u> applies to the probabilities when tossing of a die (1/6) or a coin (1/2).**
2. **<u>The frequentist</u> applies to analyses of historical observation sets (to derive e.g. climatologically based probabilities, make verifications and statistical interpretation schemes).**
3. **<u>The Bayesian</u>, <u>subjective</u> or <u>degree of belief</u> is used by e.g. weather forecasters to summarize or update their preliminary assessment considering new available information.**

Classical probabilities

rain 30% dry 70%



Ensemble of ∞ independent NWP

Selecting three balls yields

● ● ● 3% of cases

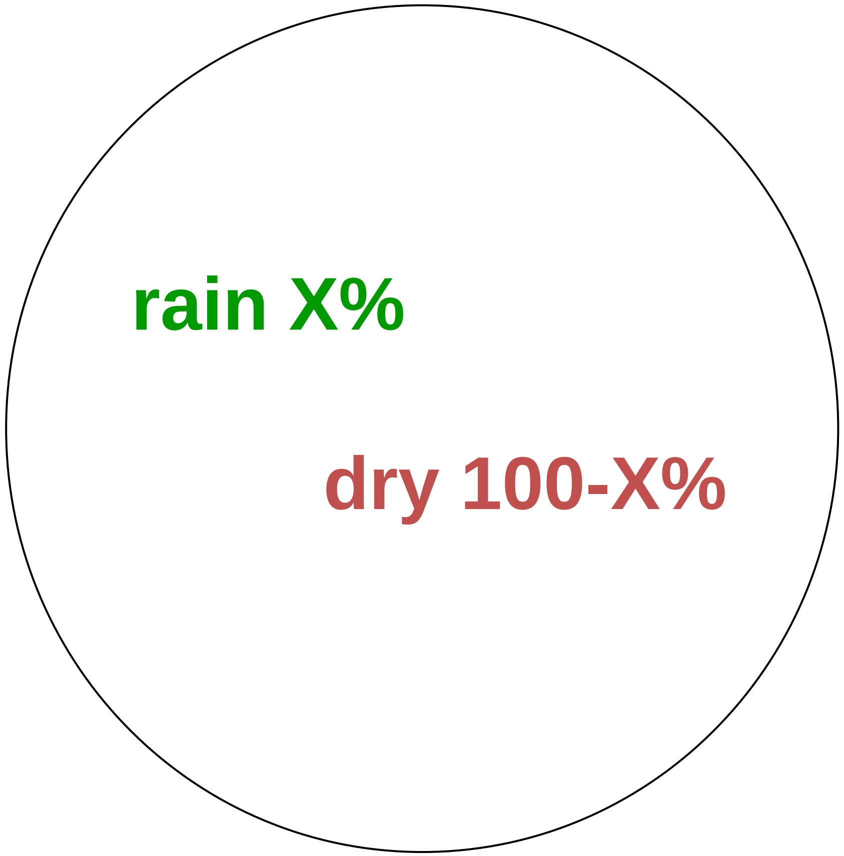
● ● o 19% - " -

● o o 44% - " -

o o o 34% - " –

. . . with a risk of 56% of misrepresentation

…on the other hand:



Ensemble of ∞ independent NWP

Selecting three balls and getting

● ● o

What does that tell us about the proportions

X and 100 – X?

This will be discussed on Wednesday!

I.2.2 Why did it take so long for probability theory to develop?

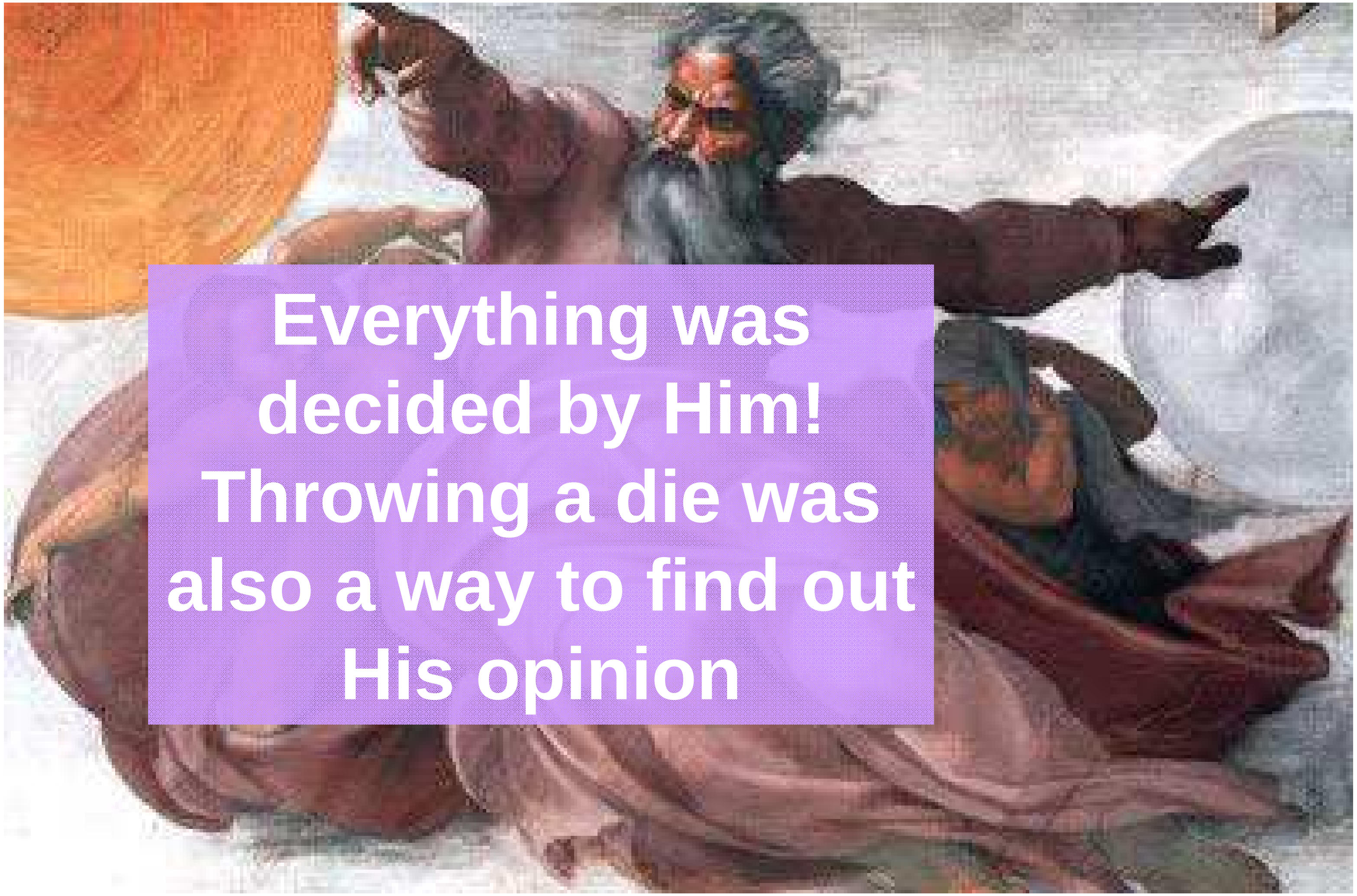
Probability theory grew out of the interest in gambling

But people have gambled since the last ice age or even before that – so why the delay??

Why did this knowledge not “spill over” into science?

Because people did not have any perception of *randomness*

(except perhaps Cicero and some other Romans)



Birth of classical probability theory

Gerolamo Cardano
1501-76

Galileo Galilee
1564-1642

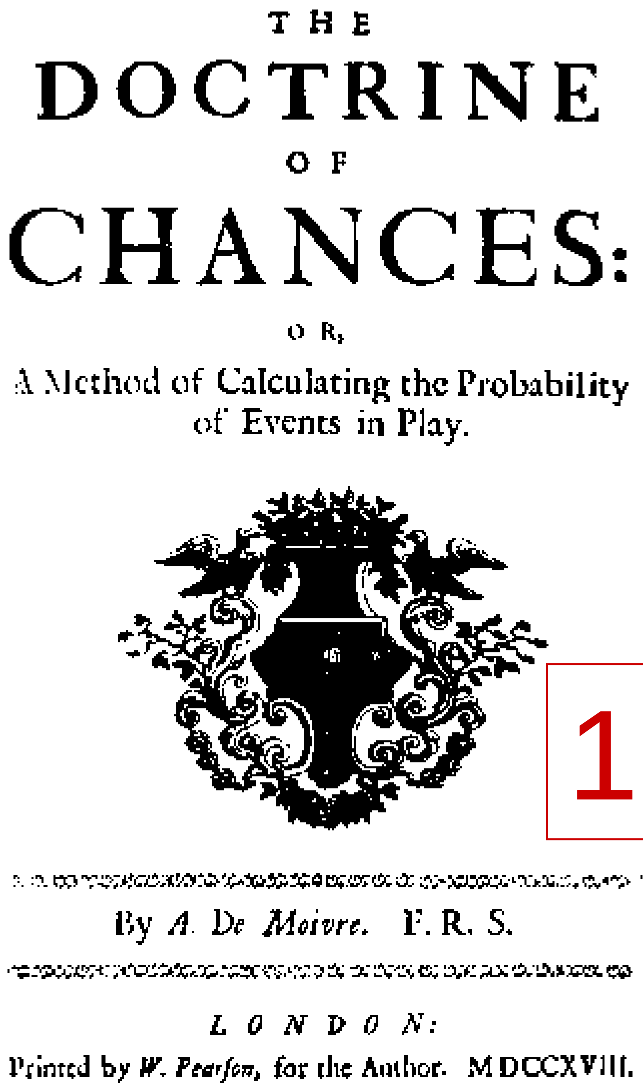
The sum of three dices seem to sum equally for 9 and 10

The last throws can be differently combined

1+2+6=9	**6**	1+3+6=10	**6**
1+3+5=9	**6**	1+4+5=10	**6**
1+4+4=9	**3**	2+2+6=10	**3**
2+2+5=9	**3**	2+4+4=10	**3**
2+3+4=9	**6**	2+3+5=10	**6**
3+3+3=9	**1**	3+3+4=10	**3**

10 is slightly more likely because 3-3-3 can only come in one version, 3-3-4 in three

Abraham De Moivre
1667-1754



THE
DOCTRINE
OF
CHANCES:
OR,
A Method of Calculating the Probability
of Events in Play.

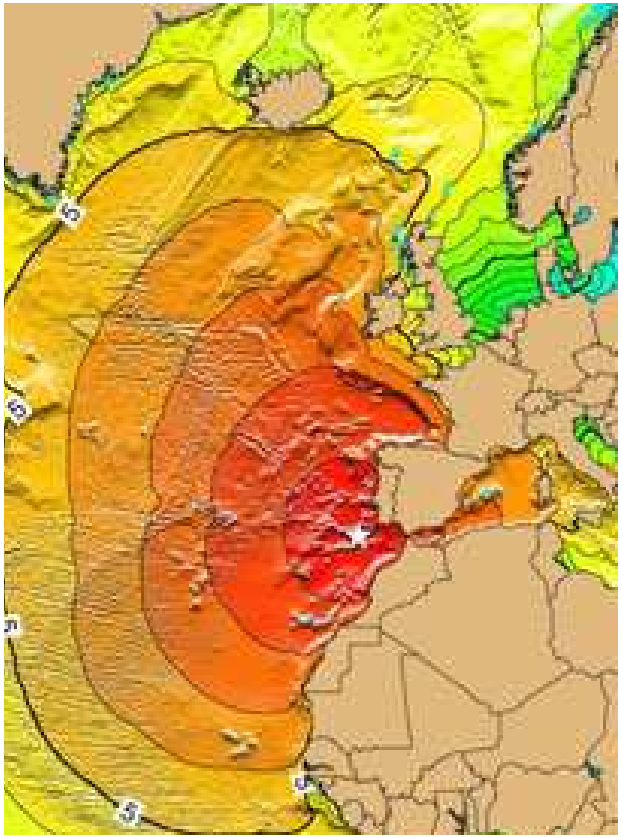
By *A. De Moivre.* F. R. S.

LONDON:
Printed by *W. Pearson,* for the Author. MDCCXVIII.

1718

From causes to effects
Deduction
Direct probabilities
Combinatorics

The Lisbon earthquake and tsunami 1755



made people start doubt the existence of an all mighty God that decided everything.

From 1750's ideas about randomness in science

I.2.3 Different degrees of determinism is still around

Five types of determinism (after Gigerenzer et al 1989,1995)

1. **<u>Metaphysical determinism:</u> Everything is already decided and there is no free will**

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3. <u>Scientific determinism:</u> We can predict the future as perfectly as we wish (if Only the Council gives us money to new computers)

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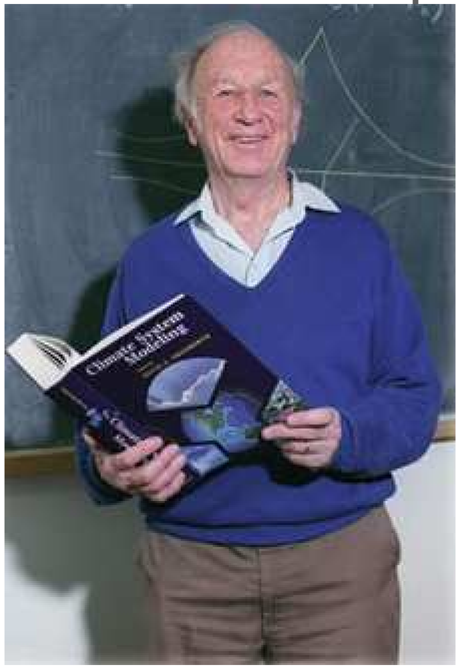
4. <u>Methodological (or pragmatic) determinism:</u> Perfect forecasts are not possible because of unknown factors



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5. Effective determinism: Although Heisenberg's uncertainty relation excludes forecasts on a molecular level it is less true at larger scales

I.2.4 Humans tend to underestimate the power of randomness and try to find ***<u>causes</u>***

a) Regular patterns can deceive:

Tossing a die five times, then **25216** is regarded as a random sequence but not the outcome **12345**

But both are equally likely! As is **66666**!

Humans have difficulties to realise that random processes can generate regular patterns

b) Winning in the National Lottery (in Britain)

Balls numbered from 1 to 99

The first 49 numbers of the National Lottery in Great Britain (once a week)

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

The number of times the first 49 numbers of the National Lottery came up winning

11	16	12	15	21	14	15
9	12	11	17	13	11	16
13	**16**	16	13	9	**20**	12
15	12	10	17	15	10	12
13	15	13	13	14	14	10
11	9	15	4	13	16	15
14	23	19	15	13	17	11

On average 14, three below 10 and three above 20

c) More about the power of randomness

The chance of two persons in a group having the same birthday (not year) being 50% is fulfilled already for a group of 23 people.

$$\begin{aligned}\bar{p}(n) - 1 \times \left(1 - \frac{1}{365}\right) \times \left(1 - \frac{2}{365}\right) \times \cdots \times \left(1 - \frac{n-1}{365}\right) \\ = \frac{365 \times 364 \times \cdots \times (365 - n + 1)}{365^n} \\ = \frac{365!}{365^n (365-n)!} = \frac{n! \cdot \binom{365}{n}}{365^n} = \frac{{}_{365}P_n}{365^n}\end{aligned}$$

d) Winning a lottery twice

With 10 000 lottery tickets, once a week and 100 lotteries around the country at the same time, the chance that *someone* over a ten year period will win twice is 25%.

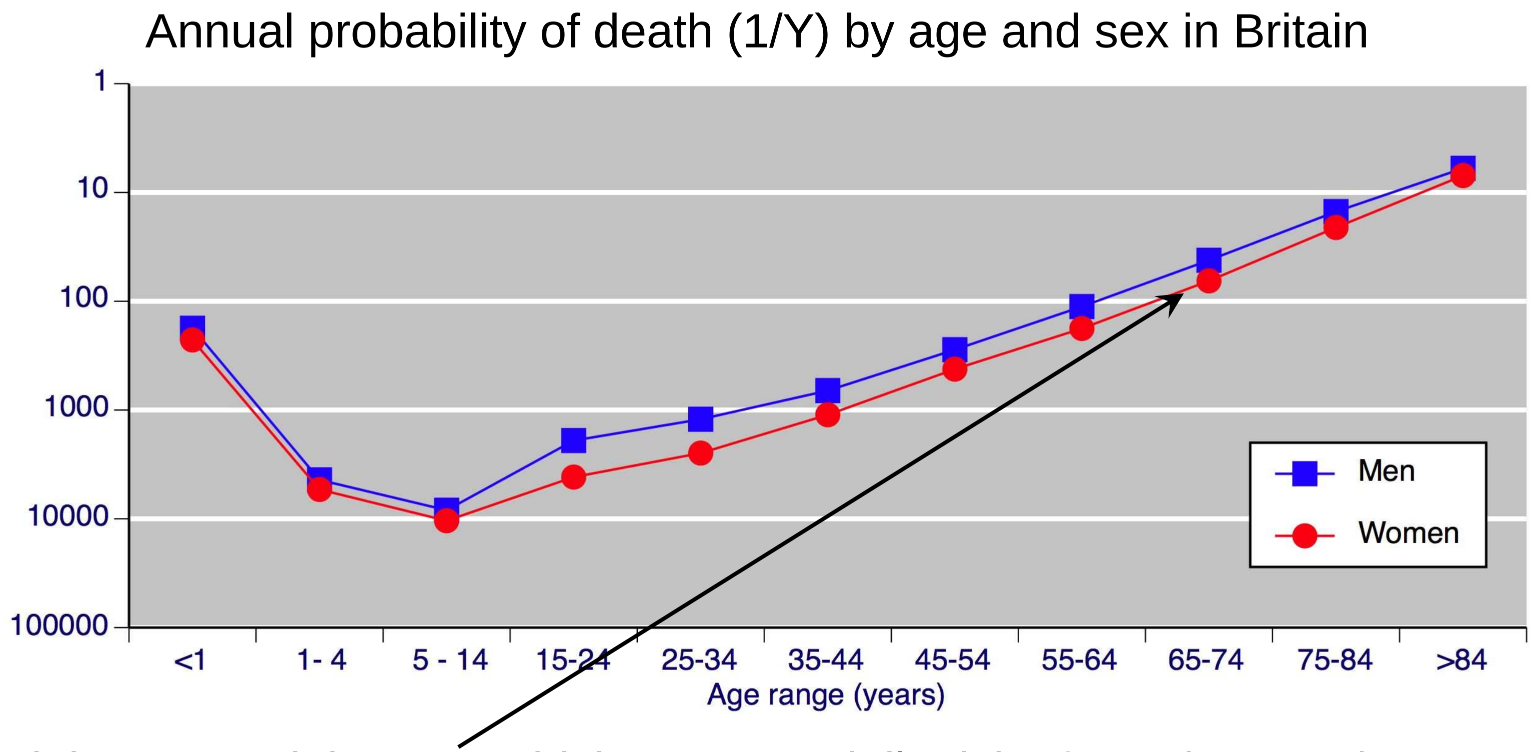
Assume person X is taking part in one lottery/week:

Probability that X will win once over ten years
$= 50 \cdot 10 \cdot 1/10\,000 = 5 \cdot 10^{-2} =$ **5%**

Probability that X will win twice over ten years
$= 25 \cdot 10^{-4} =$ **0.25%**

Probability that someone among the 100 lotteries will win twice $= 0.25\% \cdot 100 =$ **25%**

e) the probability of dying during a year



For a Brit in my age it is ≈ 2% which means a daily risk of 0.02/365 ≈ 1/20 000 or $5 \cdot 10^{-5}$

This Brit should wait until the very last to buy a 1/10 000 or 10^{-4} probability lottery ticket.

Buying a ticket three days before the draw he is more likely to be dead than to win!

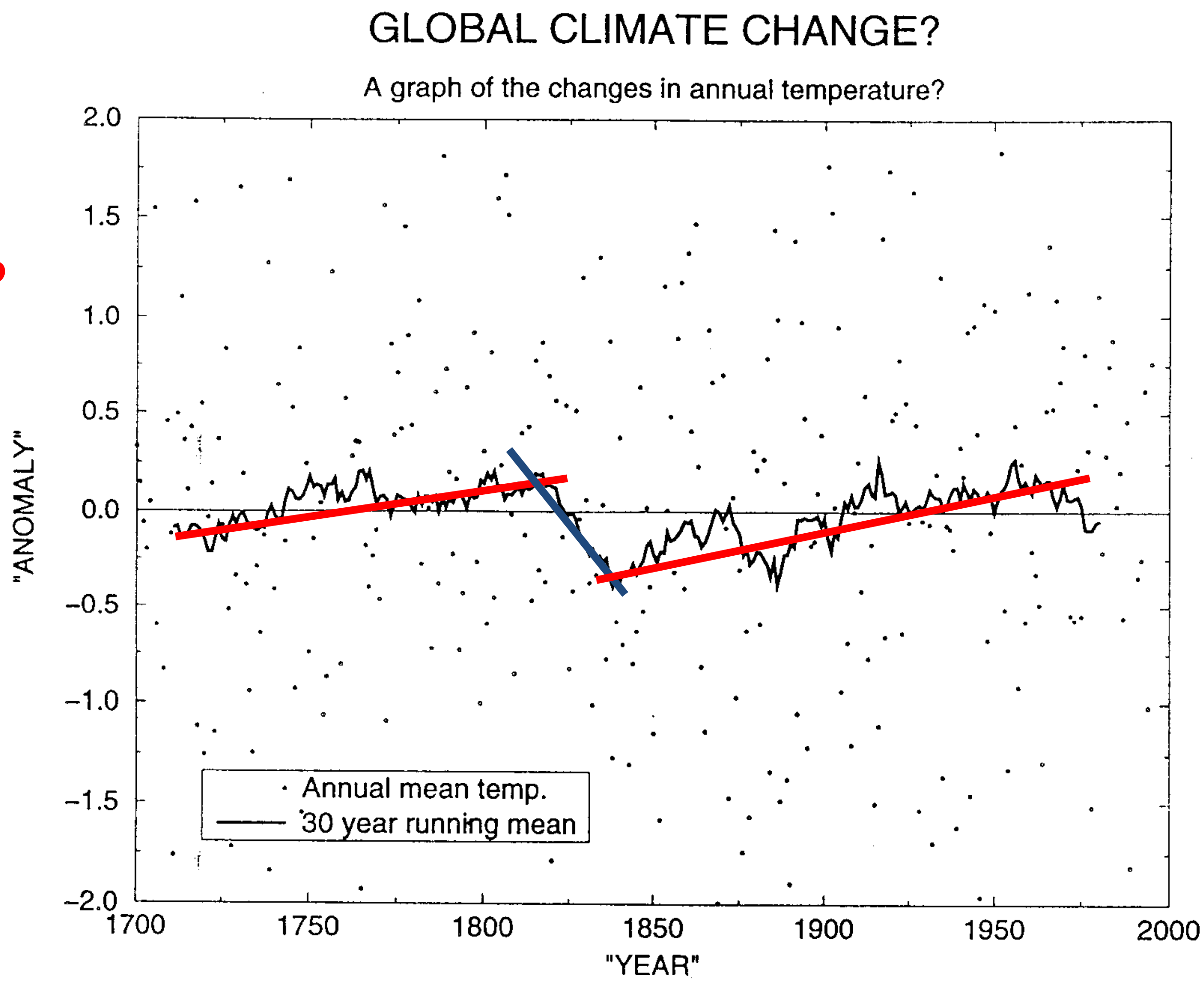
I.2.5 The Slutsky-Yule effect

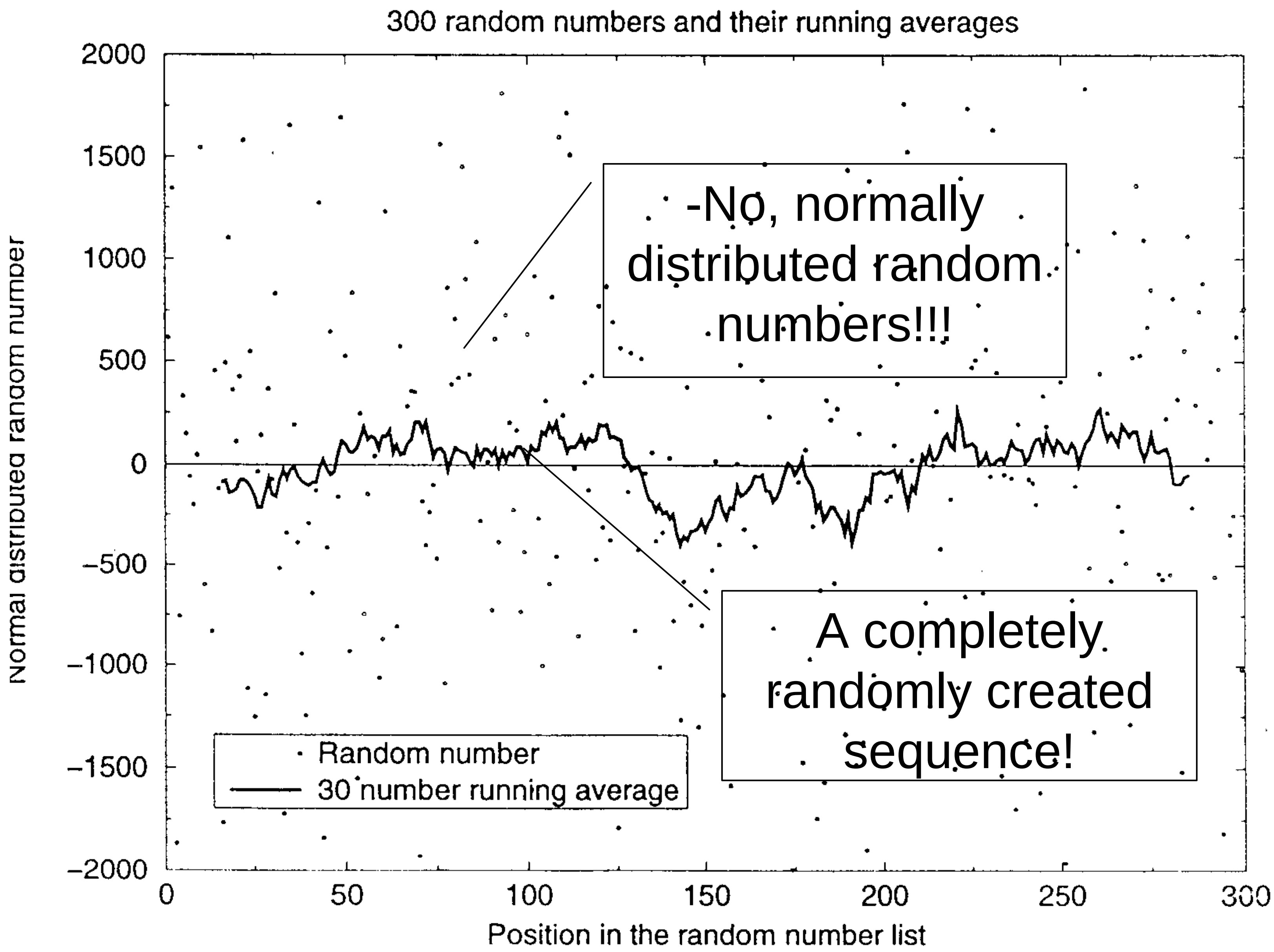
What looks like thirty year running averages of annual mean temperatures show interesting variations

What caused the warming up to 1815?

What caused the cooling thereafter?

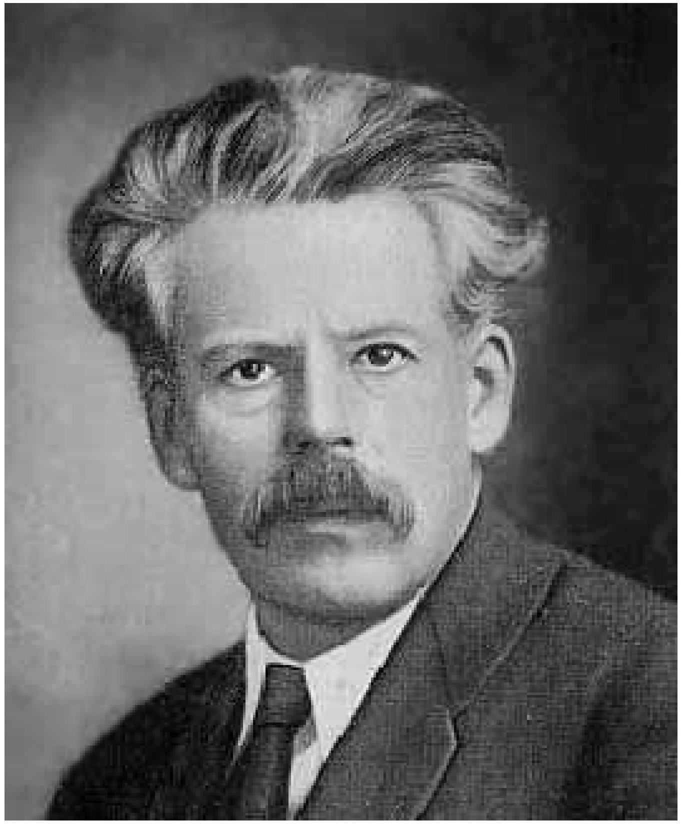
And the subsequent gradual warming by almost 0.1°/decade?




300 random numbers and their running averages
2000
1500
1000
500
0
−500
−1000
−1500
−2000
0
50
100
150
200
250
300
Position in the random number list
Normal distributed random number
Random number
30 number running average
-No, normally distributed random numbers!!!
A completely randomly created sequence!

The men behind the Slutsky-Yule Effect

Eugene Slutsky 1880-1948



USSR

George Yule 1871-1951

USA

END