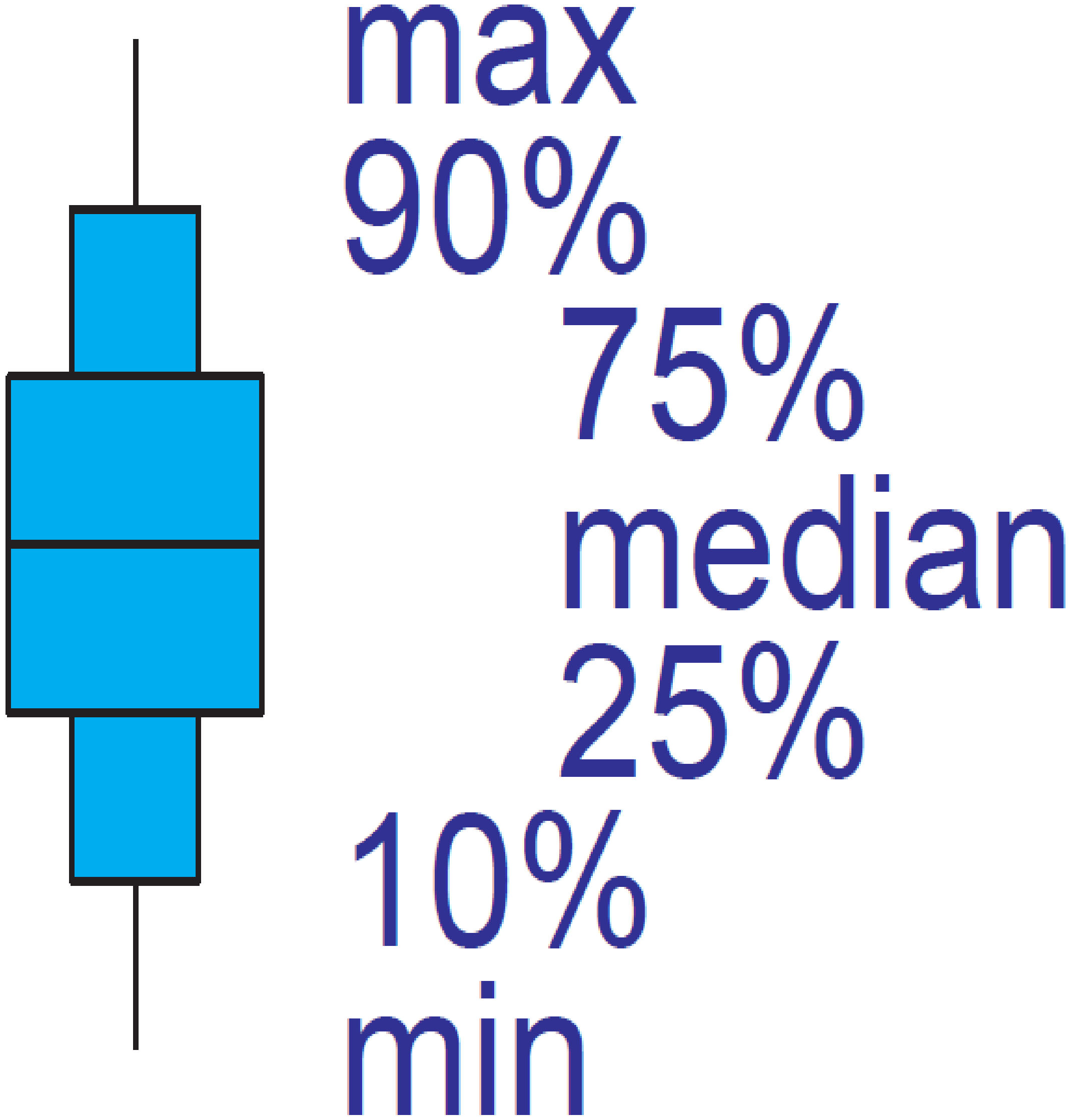
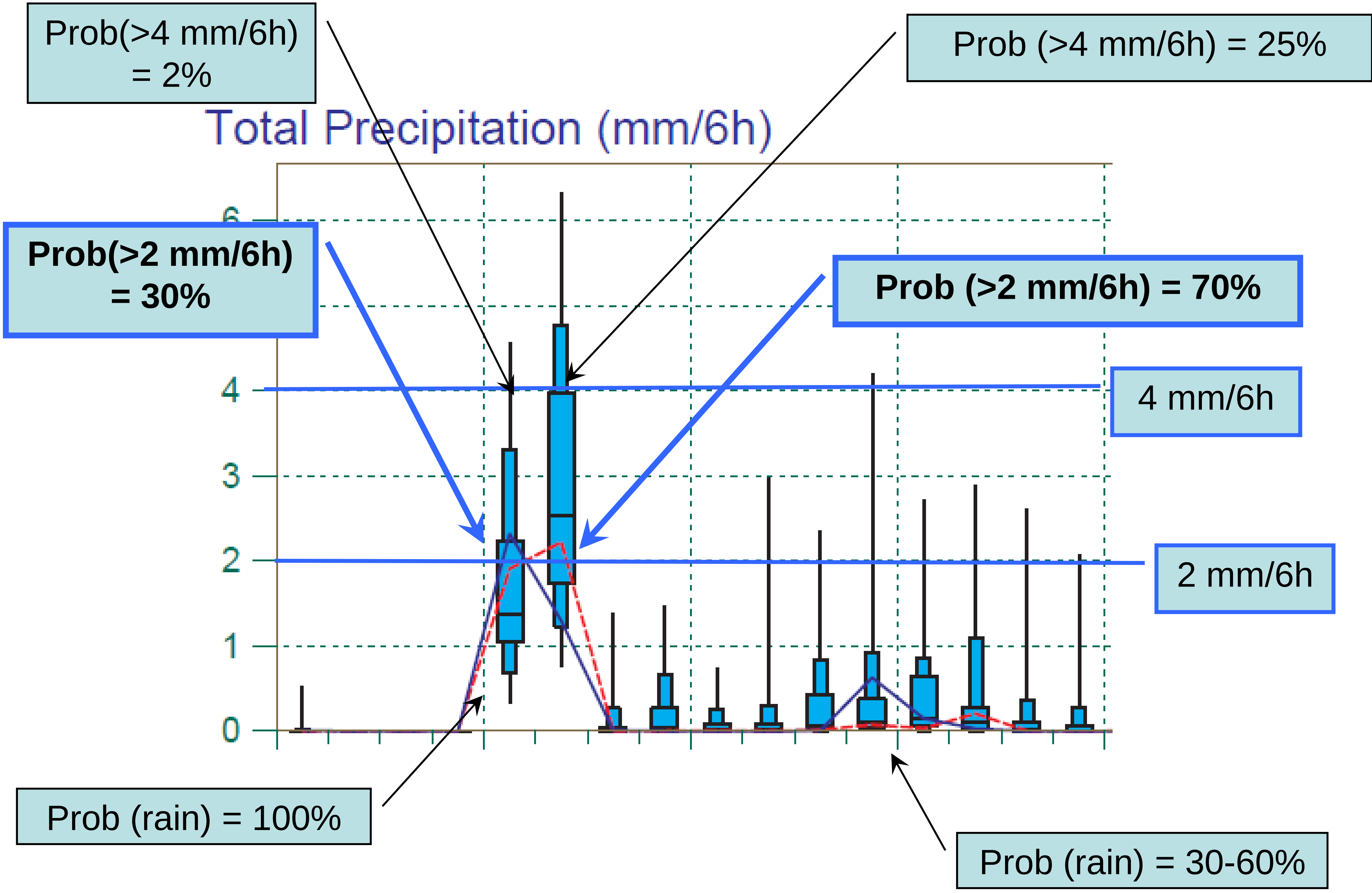
I.3 Adding or combining probabilities

I.3.1 Looking at EPS grams


max
90%
75%
median
25%
10%
min


Prob(>4 mm/6h) = 2%
Prob (>4 mm/6h) = 25%
Total Precipitation (mm/6h)
Prob(>2 mm/6h) = 30%
Prob (>2 mm/6h) = 70%
4 mm/6h
2 mm/6h
0
1
2
3
4
6
Prob (rain) = 100%
Prob (rain) = 30-60%

I.3.2 Can we add probabilities?

We can easily add probabilities if they are

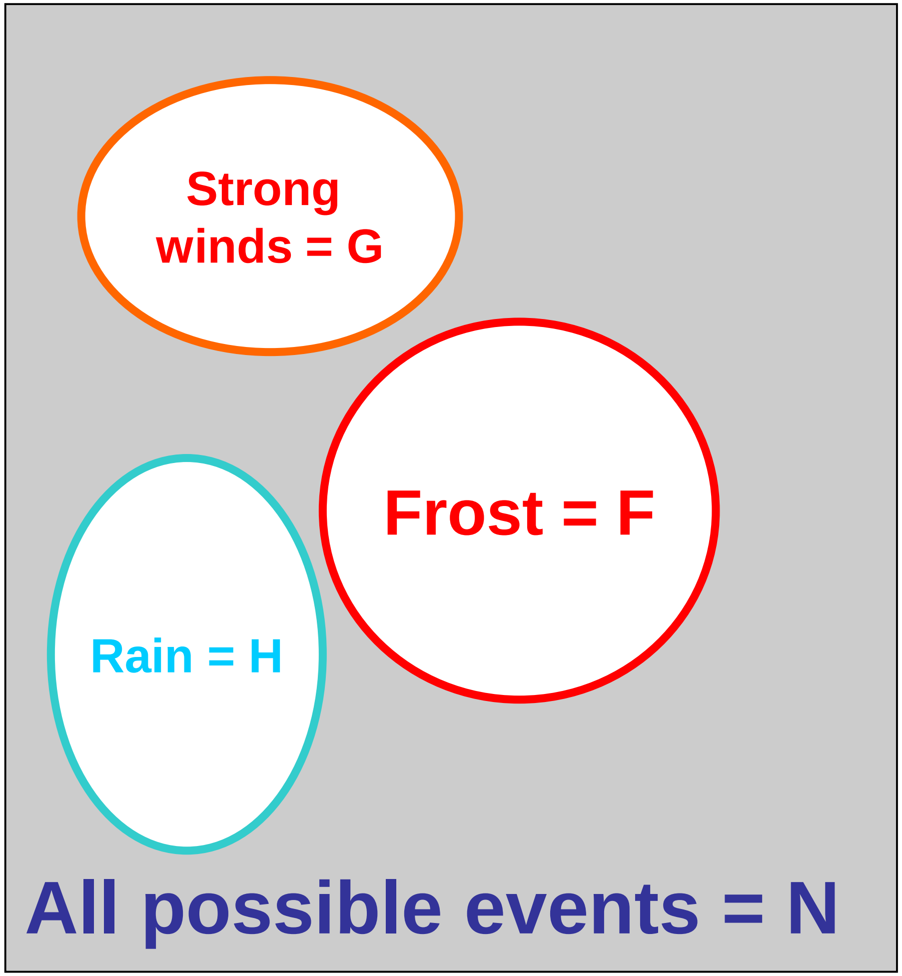
a) Exclusive

b) Independent

a) Andrei Kolmogorov's probabilities are exclusive and can easily be added

1. Probability for any event = 100%
2. Probability for one type of events = F/N
3. Probability for several mutually exclusive events = (F+G+H)/N

However, what are we after?. .



Probability for strong winds **or** rain **or** frost = **(F+G+H)/N**

Probability for strong winds **and** rain **and** frost = **0**

b) Independency:

A die is thrown twice

The chance of <u>two</u> “6” is 1/6 · 1/6 = 1/36 = **3%**

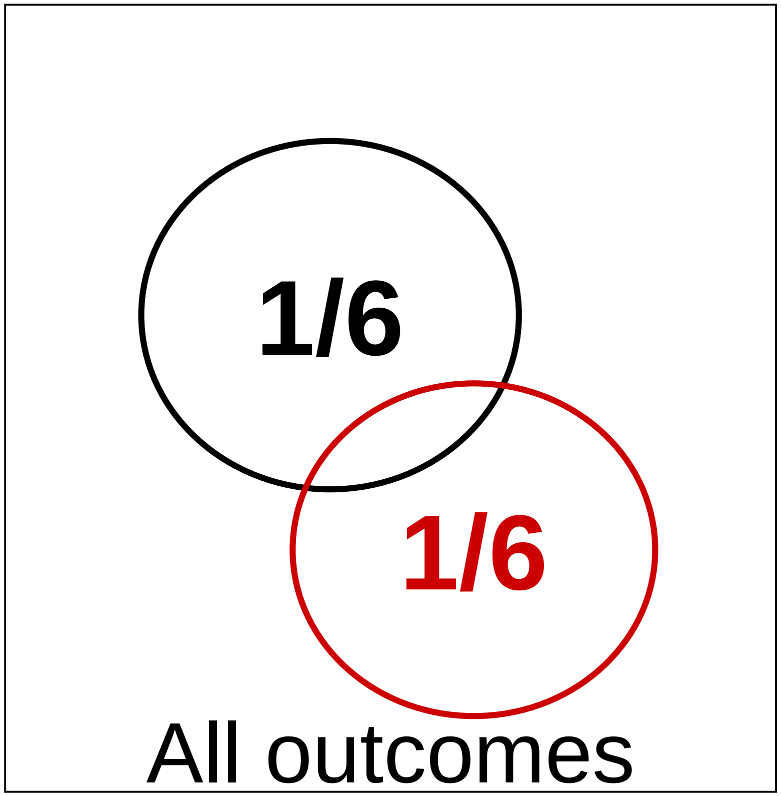
The chance of <u>no</u> “6” is 5/6 · 5/6 = 25/36 = **69%**

The chance of <u>only one</u> “6” is **2** · 1/6 · 5/6 = **28%**

6 and none **<u>or</u>** none and 6

I.3.3 To come further we must introduce the Venn diagram

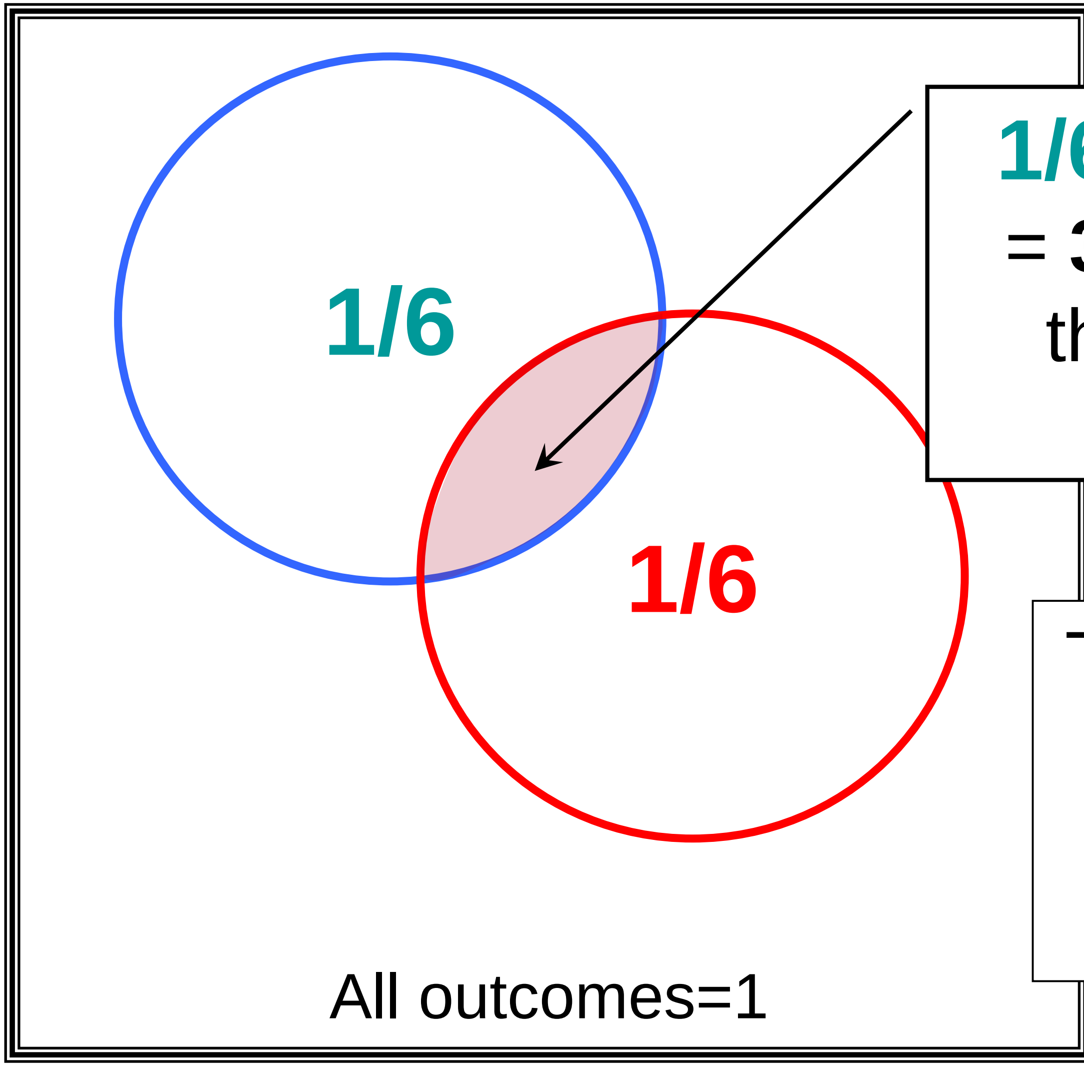
We can get some help from the "Venn diagram"



John Venn 1834-1923
Philosopher and logician

11

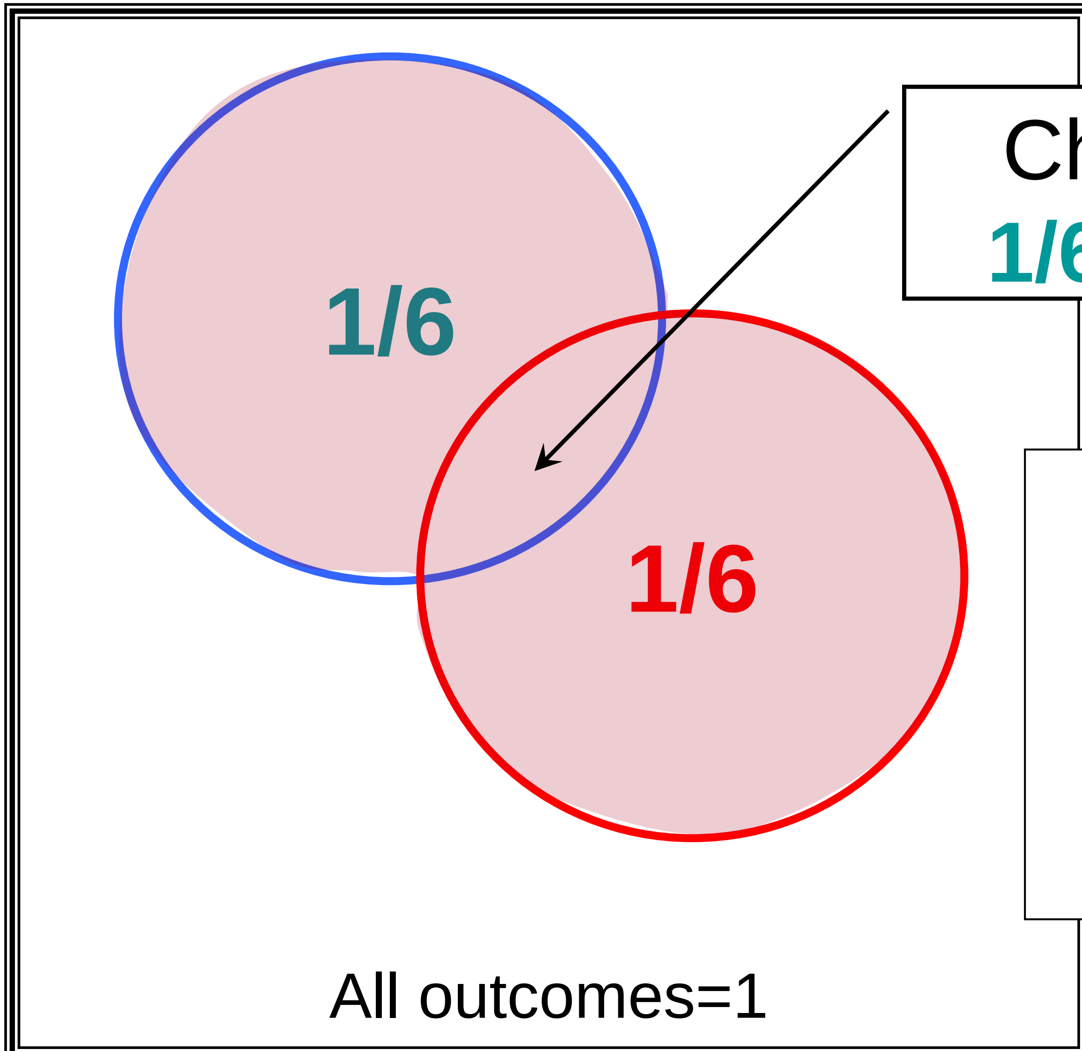
The chances of two “6” or none



1/6 of **1/6** = **1/36** = **3%** because the throws are non-correlated

The chance of not having <u>any</u> “6” is **1- (1/6**+**1/6**-**1/36)** = **25/36 = 69%**

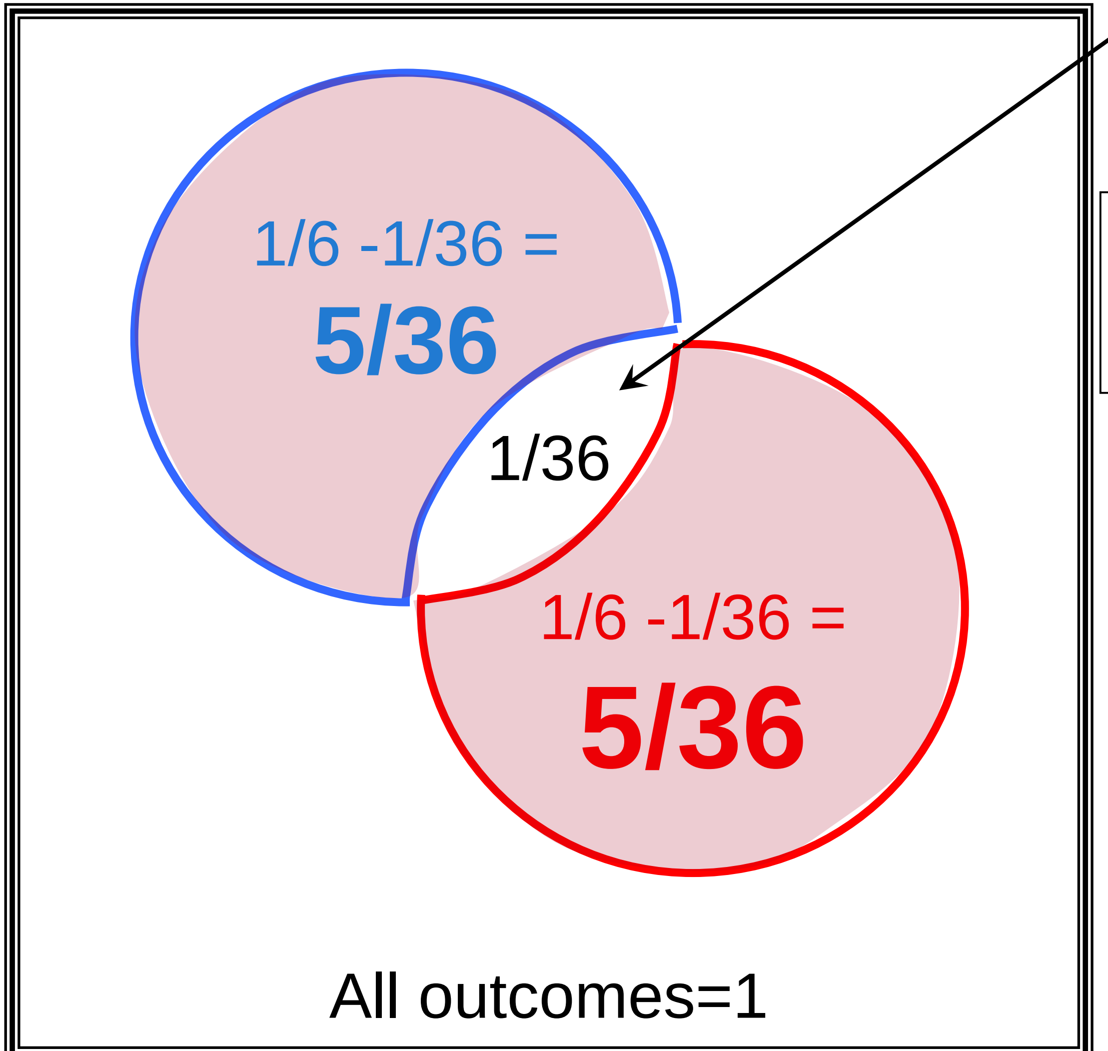
The chance of having at least one “6”



Chance of two
1/6 of 1/6 = 3%

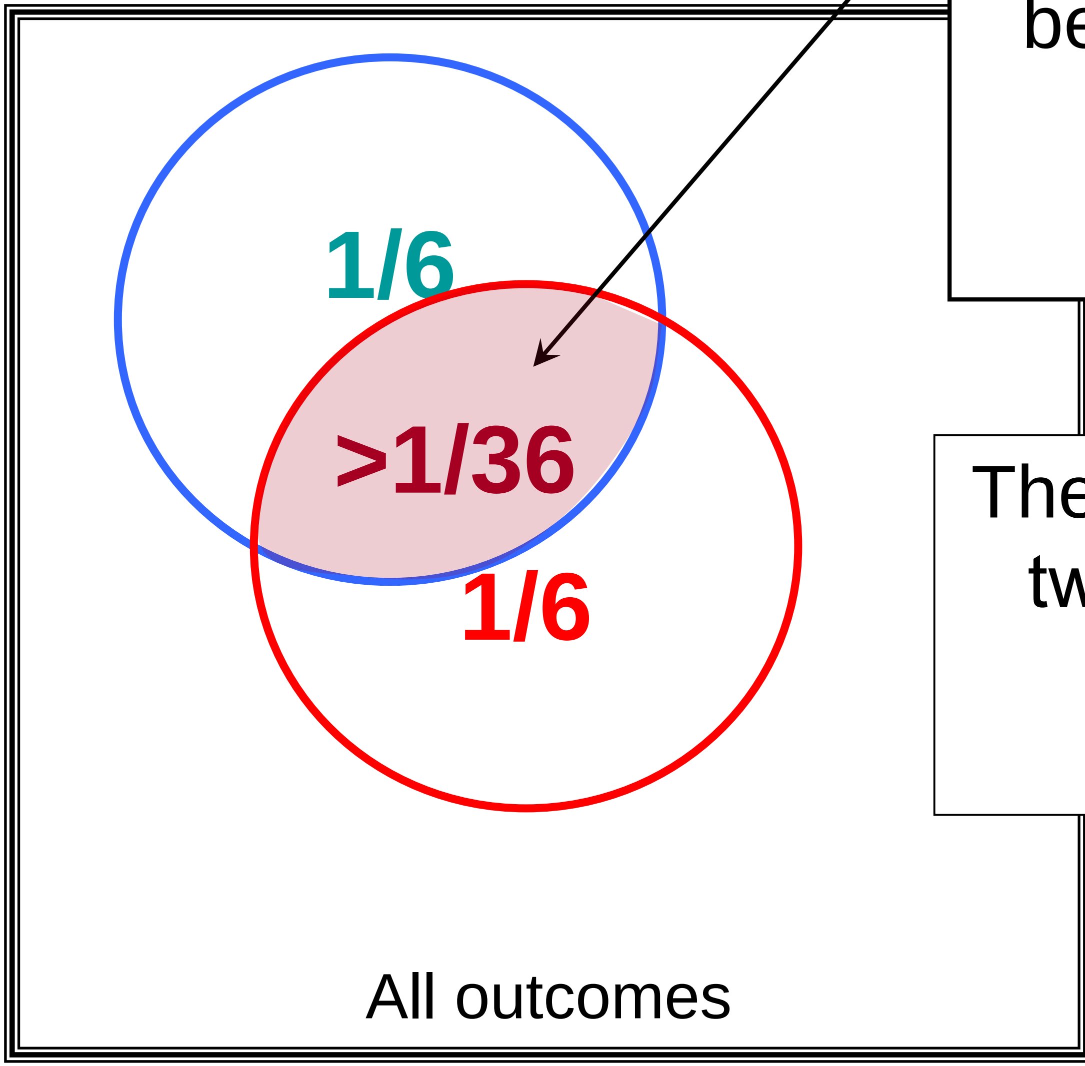
The chance of having **at least one** “6” is
1/6+1/6-1/36 =
11/36 = 31%

The chance of having <u>only one</u> "6"



The chance of two

5/36+5/36 =
10/36 = 28%



Now we are in trouble because the throws are **correlated,** dependent

The chance of having two “6” is **>1/36** and at least one “6” is **<11/36**

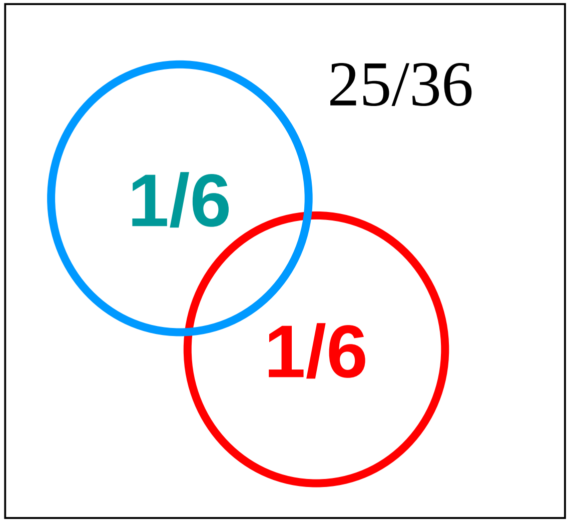
1.3.3 Correlations?

The correlation in a simple 2 x 2 table

	6	no 6
6	A	B
no 6	C	D

can easily be computed with

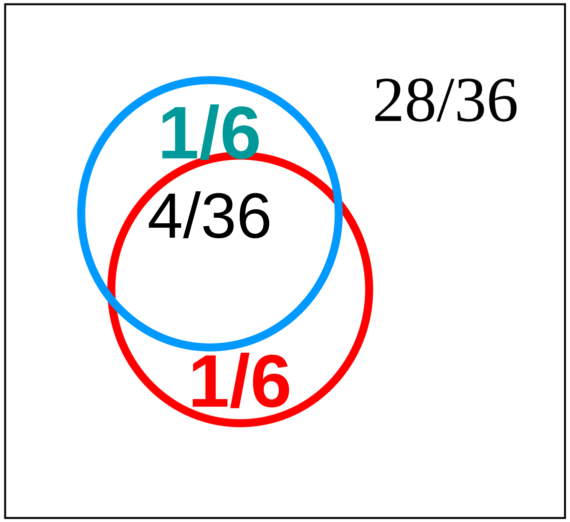
$$r = \frac{AD - BC}{\sqrt{(A+B)(A+C)(B+D)(D+C)}}$$



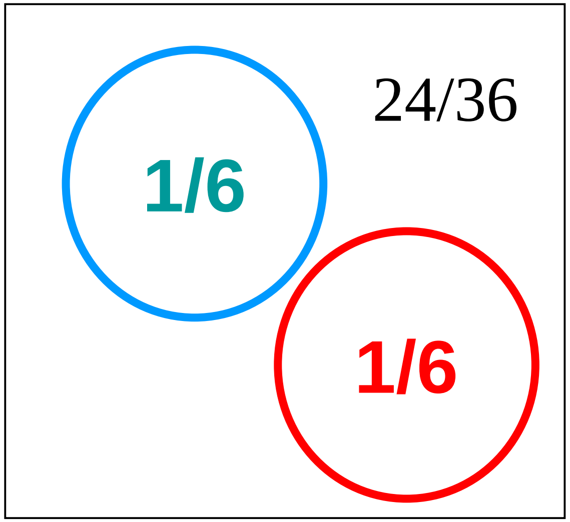
	6	no 6
6	1/36	5/36
no 6	5/36	25/36

Correlation = 0%

Correlation = 24%

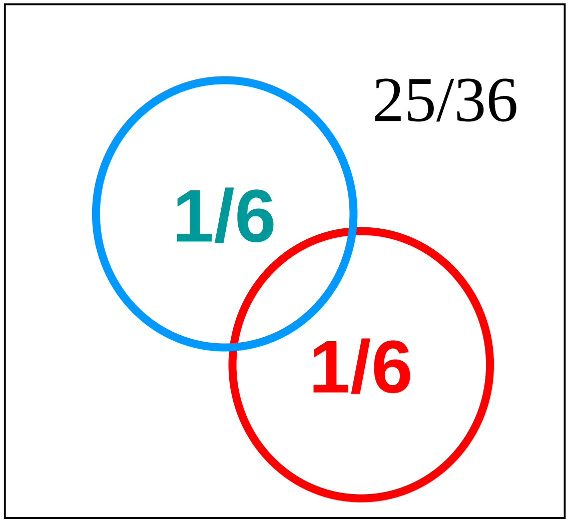


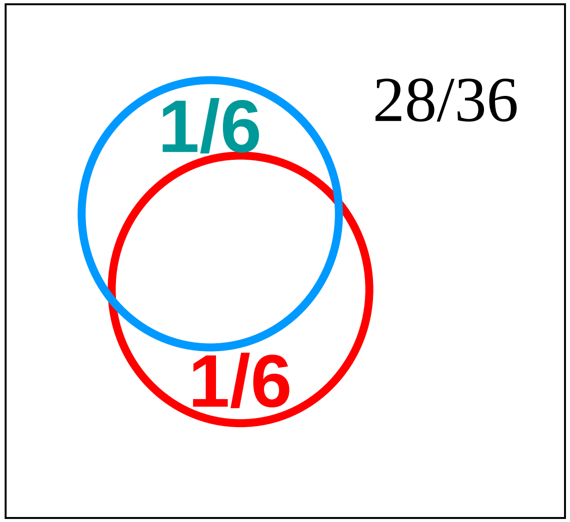
	6	no 6
6	4/36	2/36
no 6	2/36	28/36

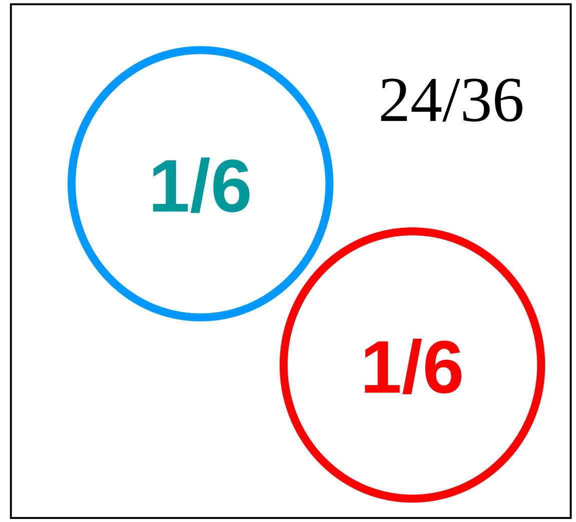


Correlation
= -25%

	6	no 6
6	0	6/36
no 6	6/36	24/36







	1/6	5/6
1/6	**1/**36	**5/**36
5/6	**5/**36	**25/**36

Correlation: 0%

	1/6	5/6
1/6	**4/**36	**2/**36
5/6	**2/**36	**28/**36

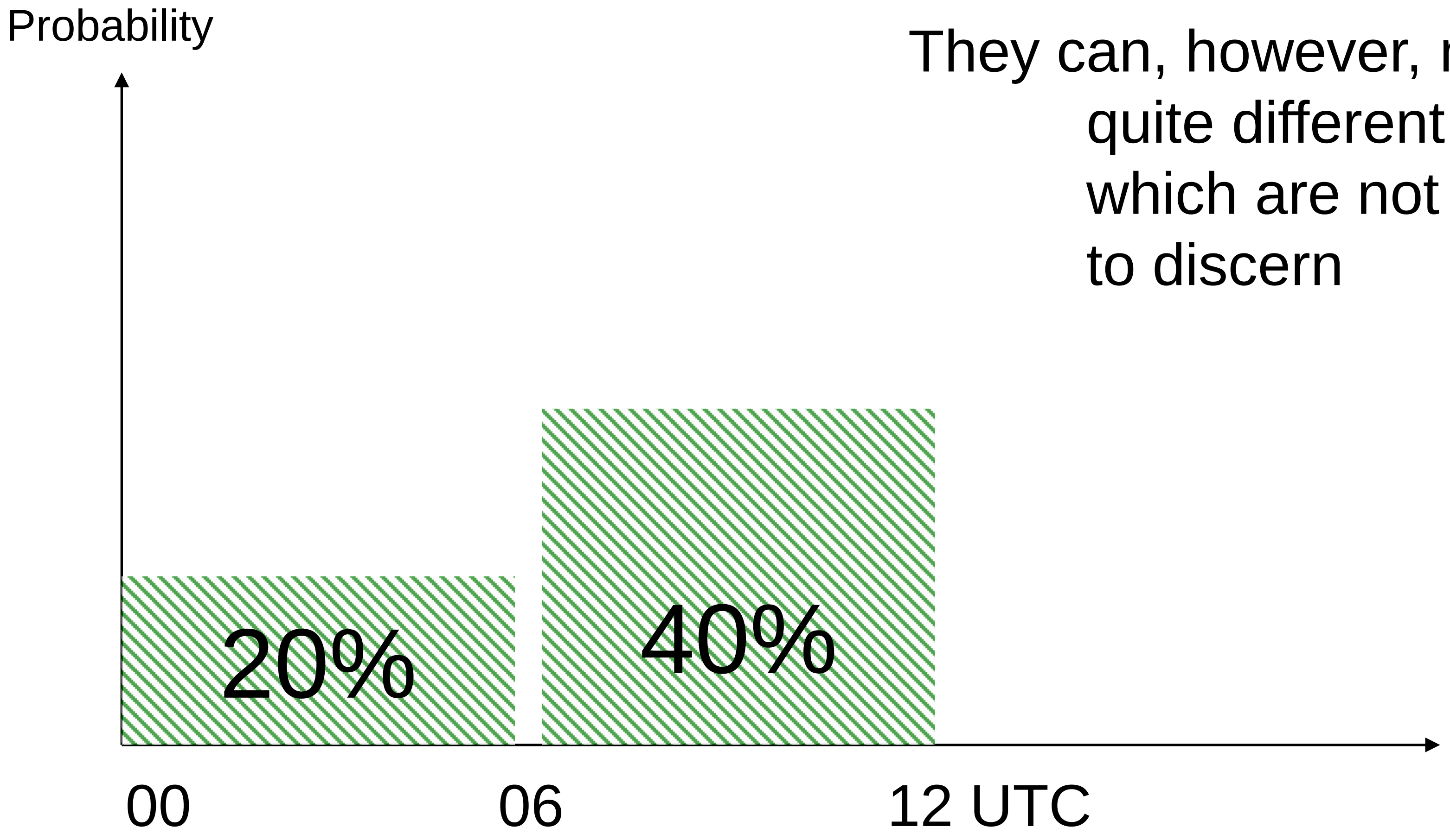
Correlation: 24%

	1/6	5/6
1/6	**0/**36	**6/**36
5/6	**6/**36	**24/**36

Correlation: -25%

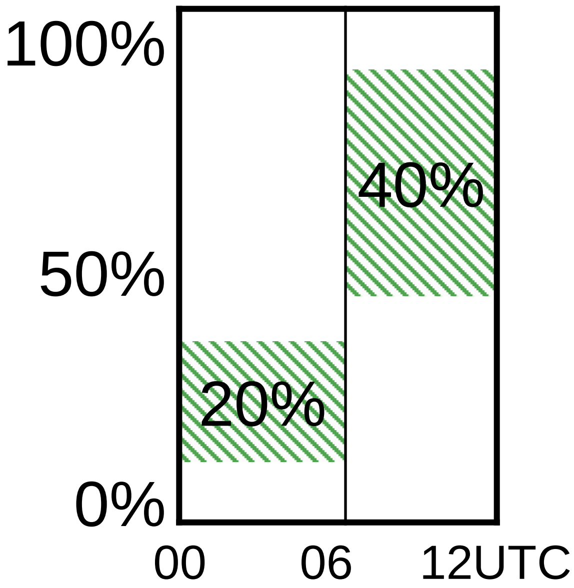
1.3.4 Real cases

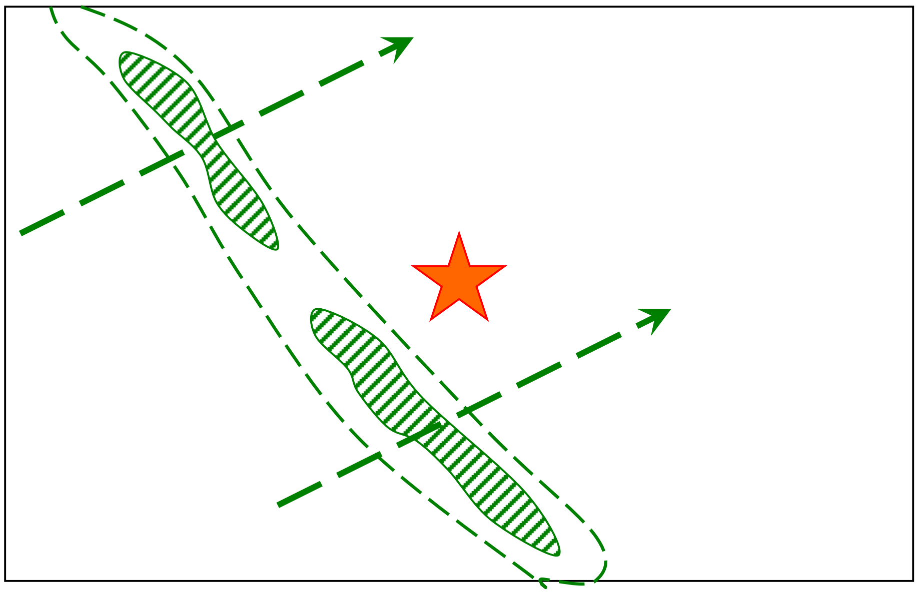
Probabilities of rain according to some reliable system



They can, however, mean quite different things which are not easy to discern

Anti-correlated time periods





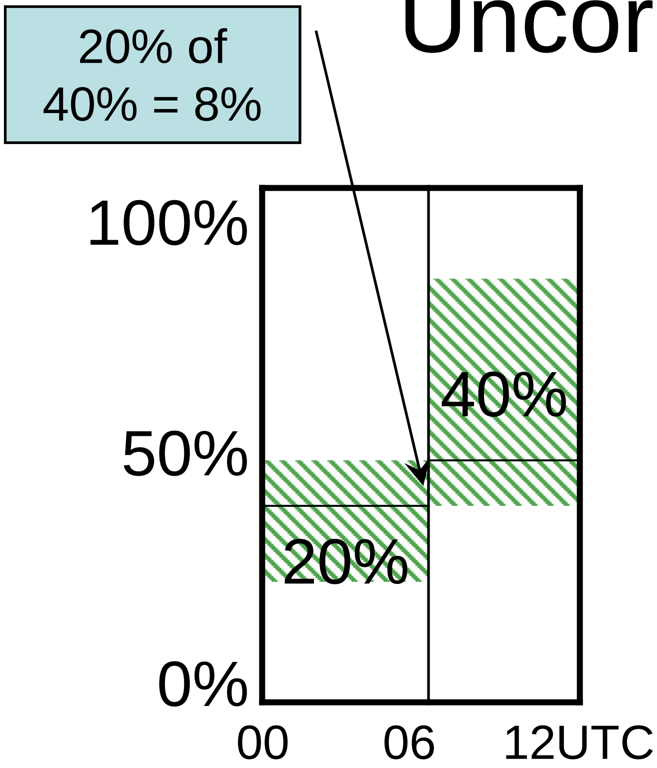
12-18UTC 06-12 UTC	R	-
R	0	20
-	40	40

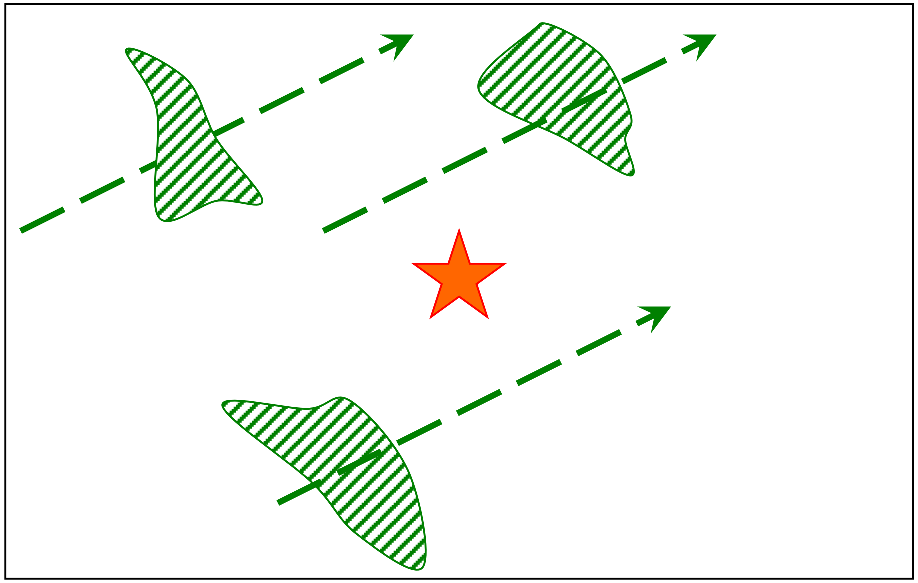
The timing is uncertain for a narrow band of rain that will pass. The total certainty is < 100% since the rain is geographically scattered

Corr = -0.20 Rain at all = 60%

Persistent rain =0%

Uncorrelated time periods





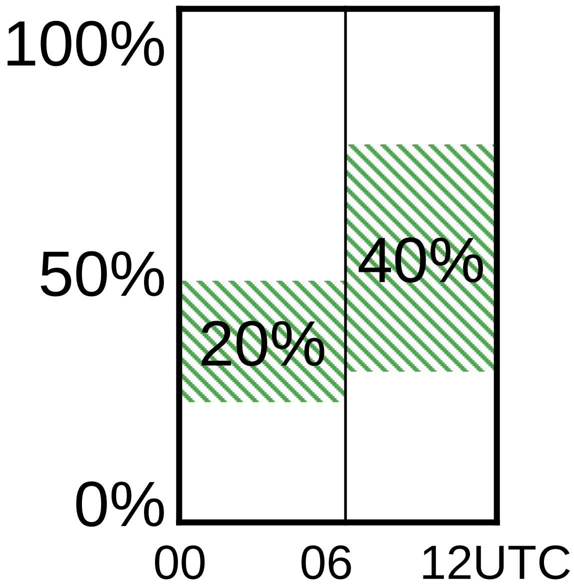
12-18UTC 06-12 UTC	R	-
R	8	12
-	32	48

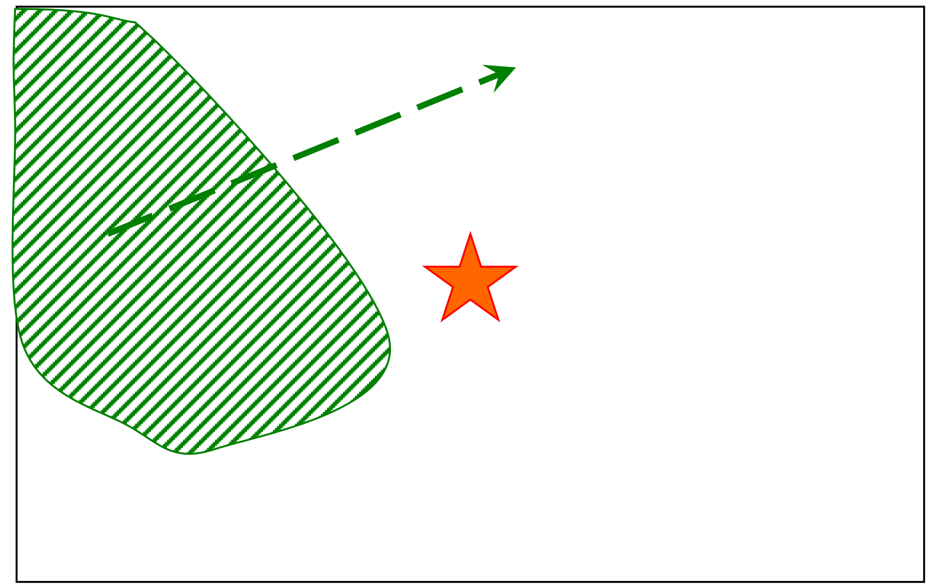
The timing is uncertain for a narrow band of rain that will pass. The total certainty is < 100% since the rain is geographically scattered

Corr = 0.0 Rain at all = 52%

Persistent rain = 8%

Correlated time periods





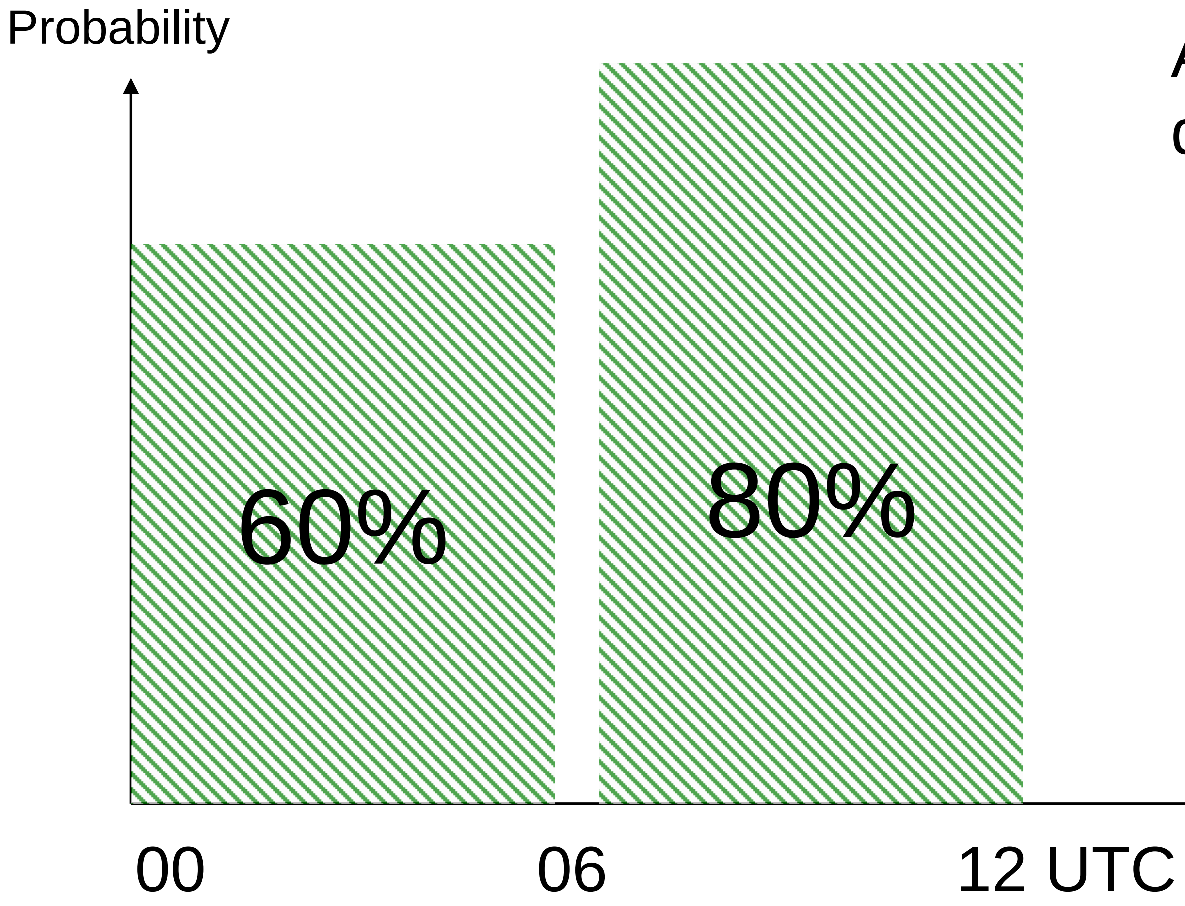
12-18UTC / 06-12 UTC	R	-
R	12	8
-	28	52

The timing is uncertain for a narrow band of rain that will pass. The total certainty is < 100% since the rain is geographically scattered

Corr = 0.65 Rain at all = 48%

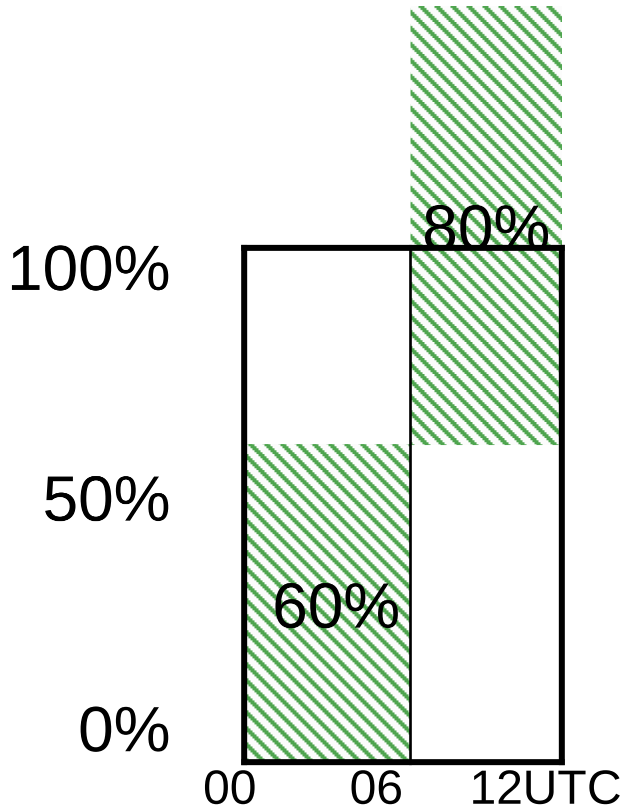
Persistent rain = 12%

Probabilities of rain according to some reliable system



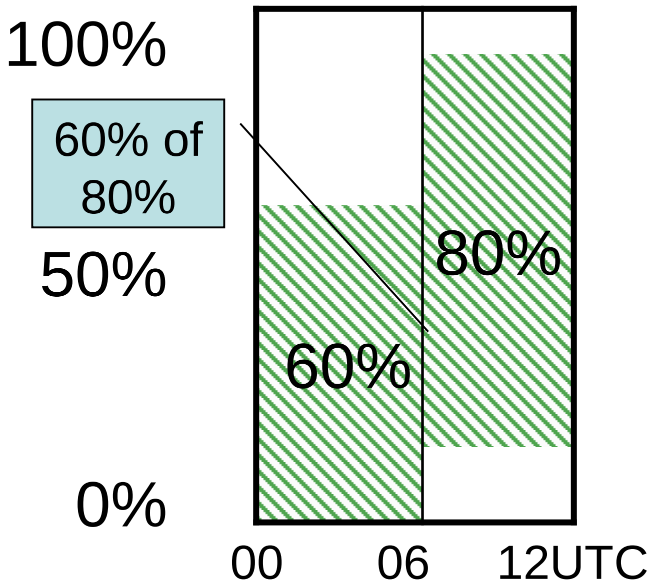
Again, they can mean quite different things

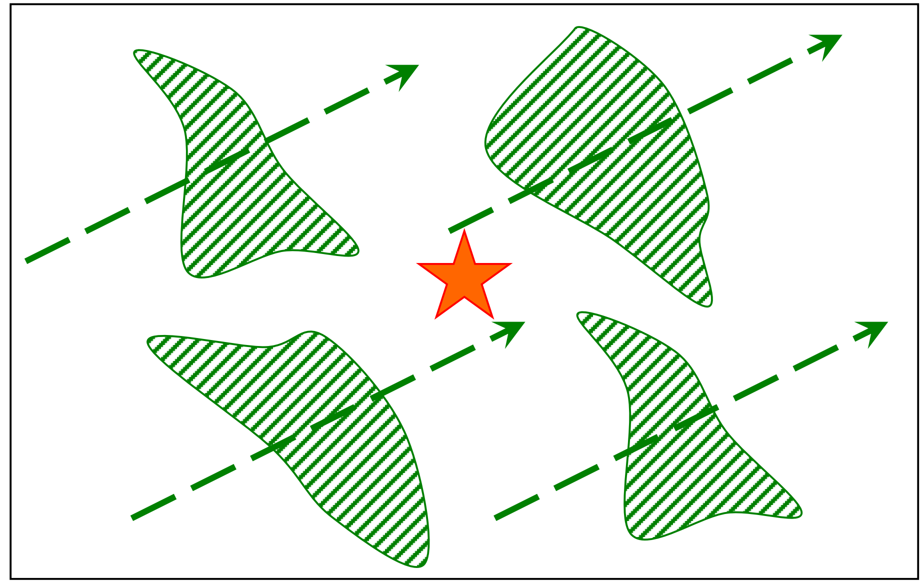
Anti-correlated time periods



Anti-correlated scenarios not possible since 0.6+0.8 >1

Uncorrelated time periods





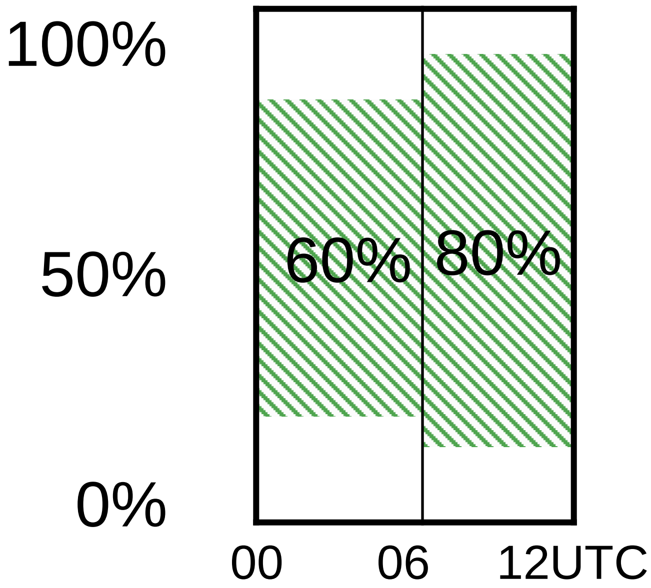
12-18UTC 06-12 UTC	R	-
R	48	12
-	32	8

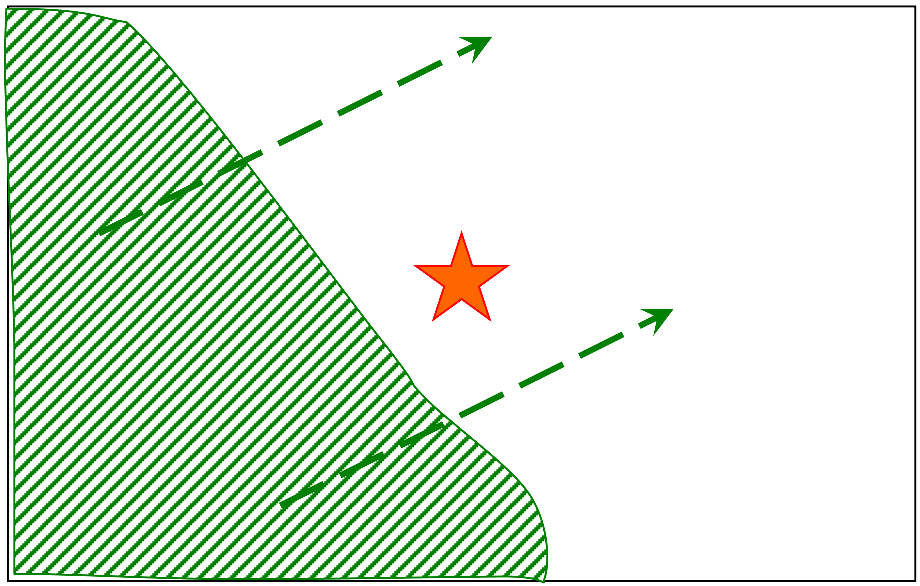
The occurrence, intensity and timing is uncertain for geographically scattered rain showers

Corr = 0.00 Rain at all = 92%

Persistent rain = 48%

Correlated time periods



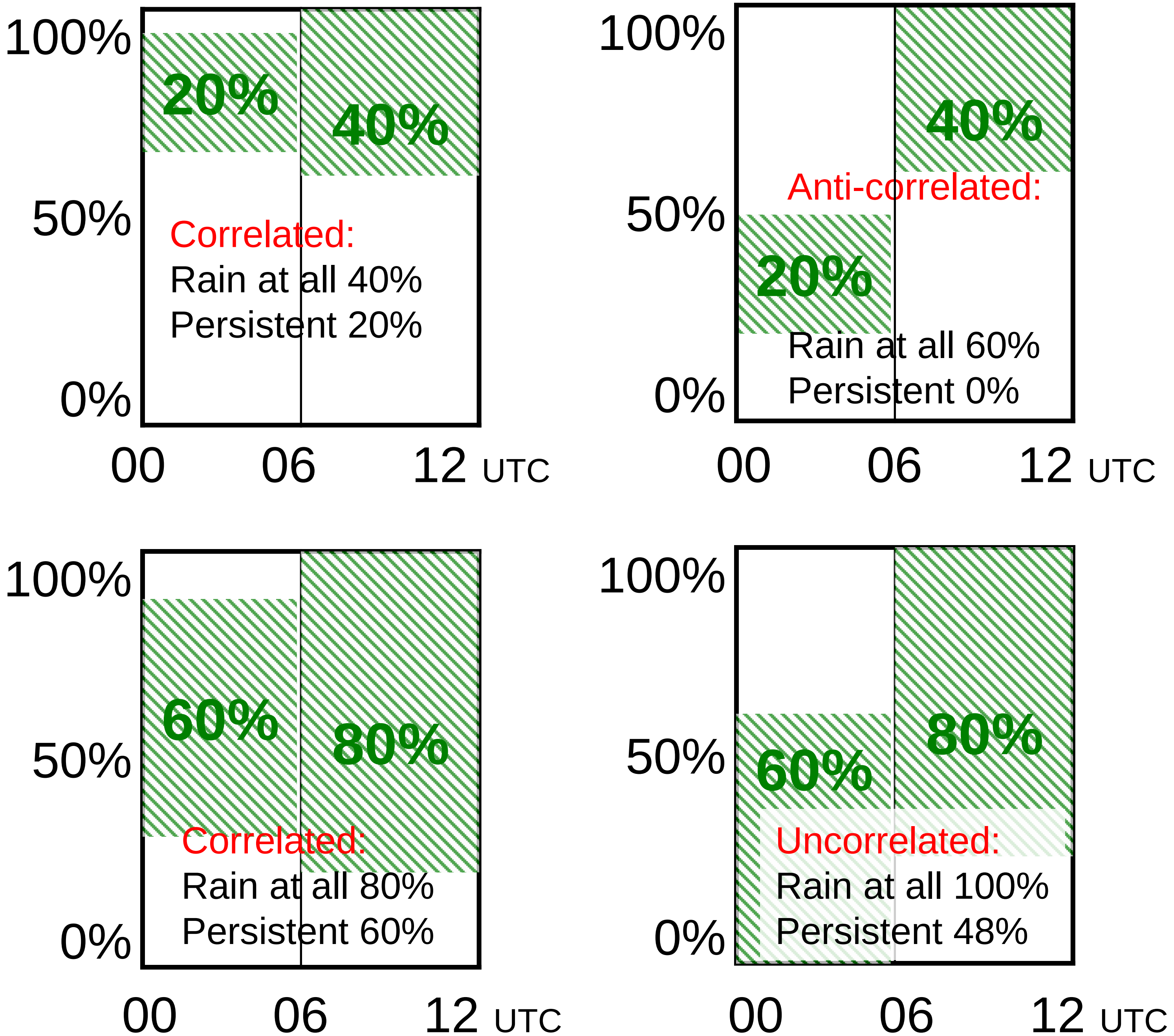


12-18UTC 06-12 UTC	R	-
R	60	0
-	20	20

The occurrence, intensity and timing is uncertain for geographically scattered rain showers

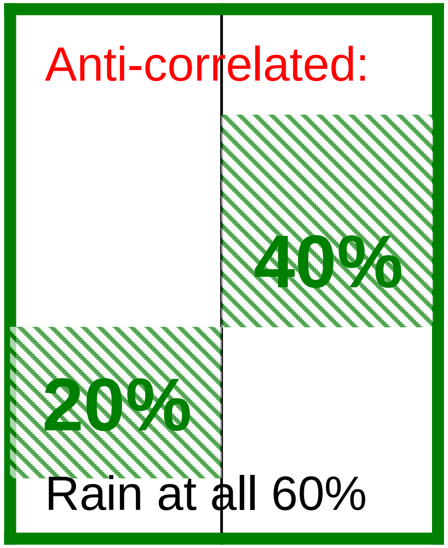
Corr = 0.61 Rain at all = 80%

Persistent rain = 60%



100%
50%
0%
20%
40%
Correlated:
Rain at all 40%
Persistent 20%
00
06
12 UTC
100%
50%
0%
40%
Anti-correlated:
20%
Rain at all 60%
Persistent 0%
00
06
12 UTC
100%
50%
0%
60%
80%
Correlated:
Rain at all 80%
Persistent 60%
00
06
12 UTC
100%
50%
0%
80%
60%
Uncorrelated:
Rain at all 100%
Persistent 48%
00
06
12 UTC

Thumb rules for rain occurring at all:

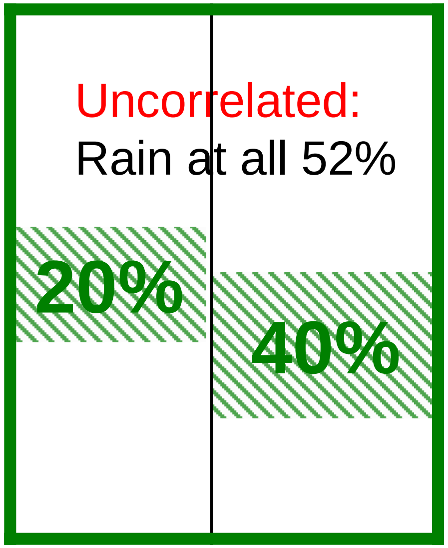


1. Anti-correlated probabilities:

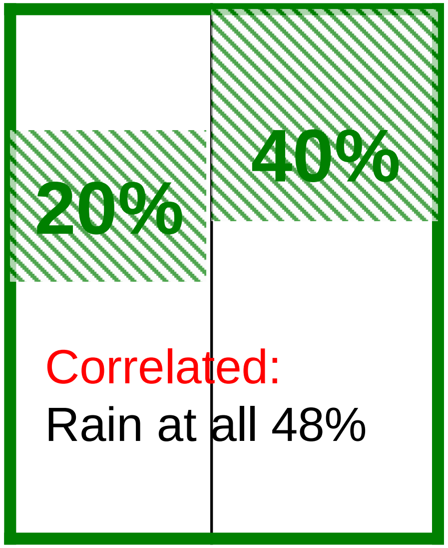
$$P = p_1 + p_2$$

2. Uncorrelated probabilities:

$$P = 1 - (1-p_1)(1-p_2))$$



Thumb rules for rain occurring <u>at all</u>:

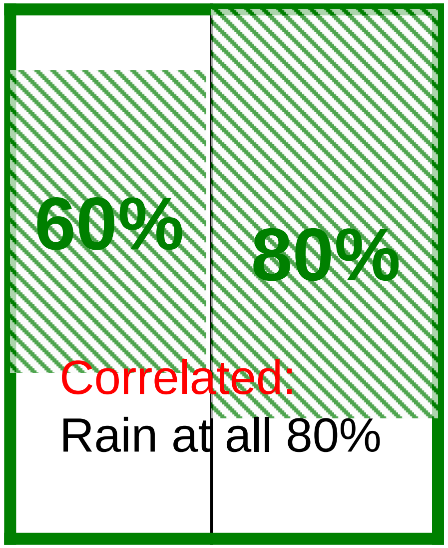


1. Anti-correlated probabilities:

$$P = p_1 + p_2$$

2. Uncorrelated probabilities:

$$P = 1 - (1-p_1)(1-p_2))$$



3. Correlated probabilities

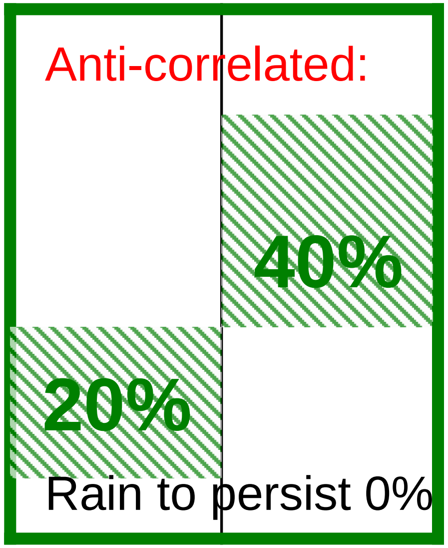
$$\mathbf{P \approx \text{the largest } (p_1,p_2)}$$

Used in "fuzzy logic" or "fuzzy set theory" (Zadeh, 1978)

Thumb rules for rain to persist:

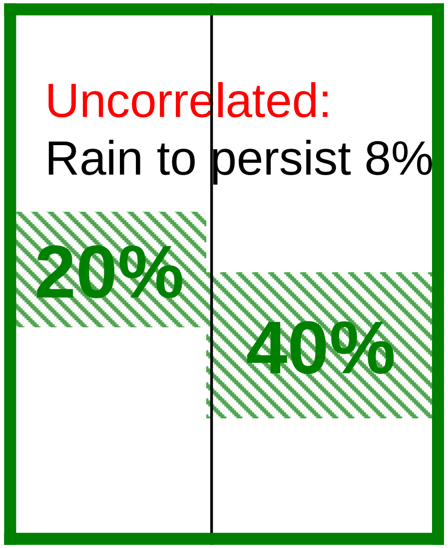
1. Anti-correlated probabilities:

$$P = 0$$

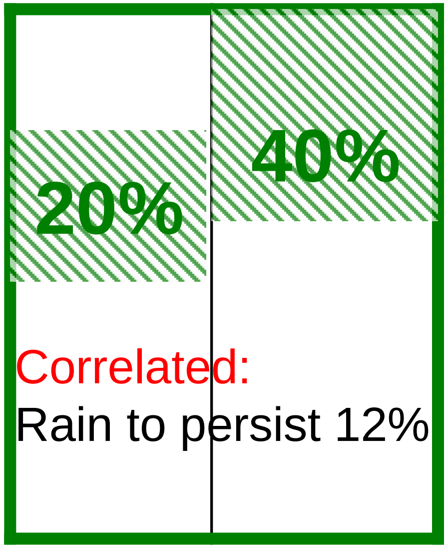


2. Uncorrelated probabilities:

$$P = p_1 \cdot p_2$$



Thumb rules for rain to <u>persist</u>:

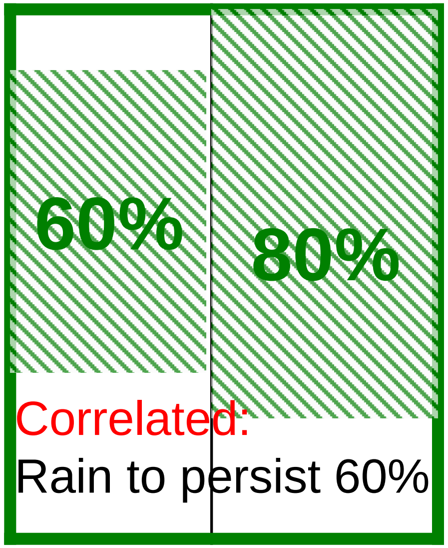


1. Anti-correlated probabilities:

   P = 0

2. Uncorrelated probabilities:

   $P = p_1 \cdot p_2$

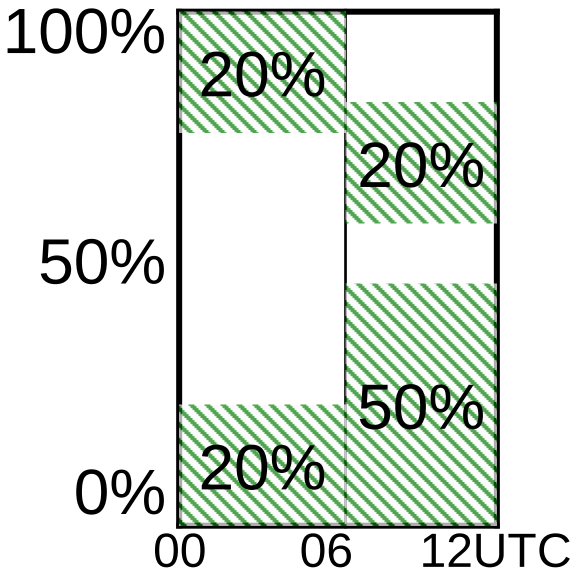


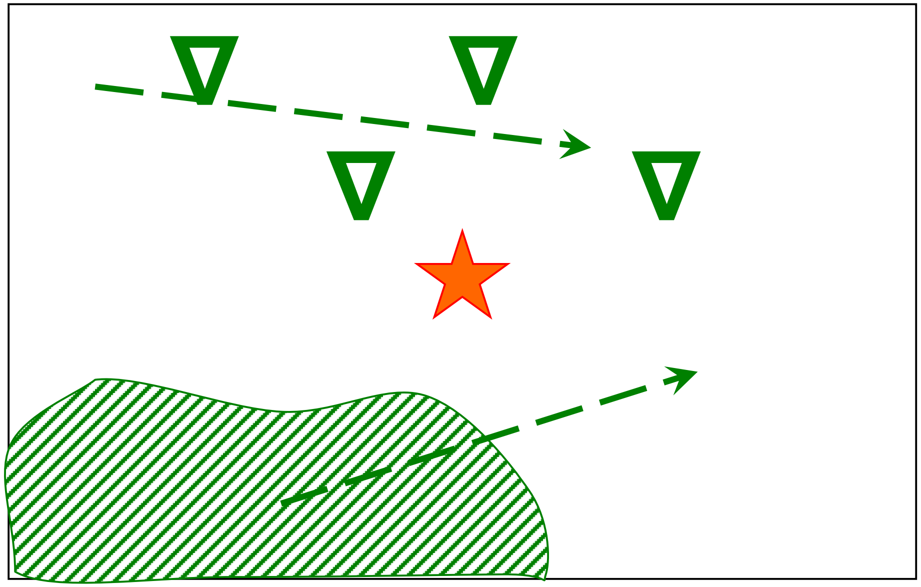
3. **Correlated probabilities**

   $P \approx$ the smallest (p_1, p_2)

Used in “fuzzy logic” or “fuzzy set theory” (Zadeh, 1978)

Anti-correlated time periods





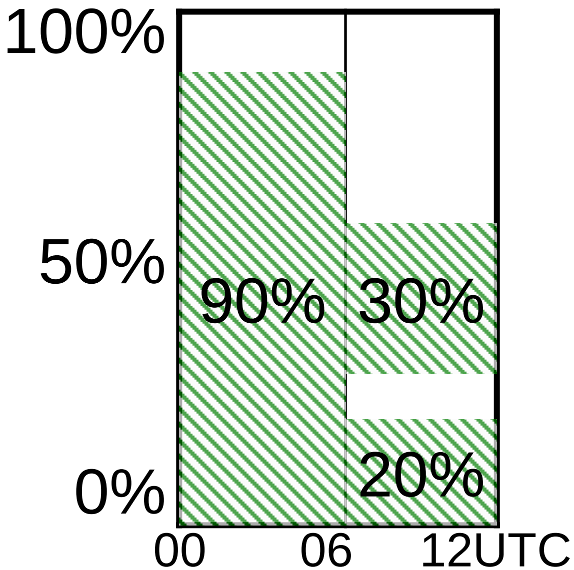
12-18UTC 06-12 UTC	R	-
R	24	16
-	46	14

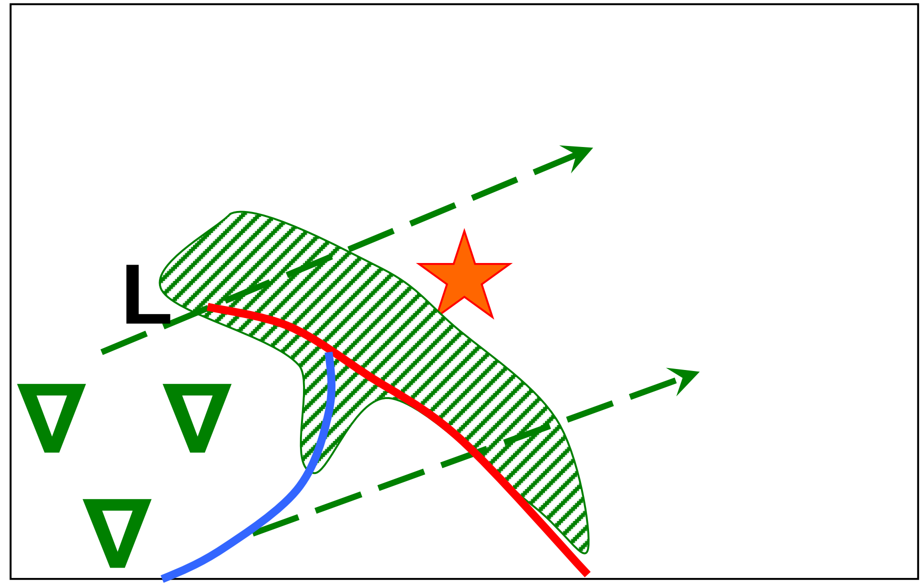
The timing is uncertain for the arrival of a major rain area. The total certainty is < 100% since there is a small risk that the rain will be delayed

Corr = -0.18 Rain at all = 86%

Persistent rain = 24%

Correlated time periods





12-18UTC 06-12 UTC	R	-
R	50	40
-	0	10

The timing is uncertain for the arrival of a major rain area. The total certainty is < 100% since there is a small risk that the rain will be delayed

Corr = 0.37 Rain at all = 90%

Persistent rain = 50%

END