I. Classical probabilities

I.4 Markov chains

1932

WEATHER RIGHT

WHILE day-to-day weather forecasting enjoys reasonable accuracy, meteorologists have still to work out a basis for long-range prophecies. Nevertheless, Dr. C. F. Marvin, head of the U. S. Weather Bureau, is experimenting with a "scientific guesser." Small balls are marked for a certain kind of weather. The balls are thoroughly mixed and poured into troughs. Their sequence, depending solely upon laws of chance, has proved strikingly similar to actual weather records.

Markov chains?

Markov chains as pedagogic, analytical and prognostic tool

1. Helps us understand probabilities and their additions and co-variations

2. Helps us analyse data in a new and interesting way

3. Might not stand up on its own, but provides a good complement to traditional statistical post-processing

A transition table for rain fall at Stockholm-Arlanda airport (100 typical cases)

62% (17/28) probability of continued rain

38% probability of rain to dry

Following 6 hours

First 6 hours	rain	dry	
rain	**17**	**11**	28 cases
dry	**11**	**61**	72 cases
	28 cases	72 cases	

15% (11/72) probability of dry to rain

85% (61/72) probability of staying dry

We note that the climatological rain probability is **28%**

Transition probabilities

62% probability of continued rain

38% probability of rain to dry

Next 6 hours

Last 6 hours

$$\begin{array}{c|cc} & \mathbf{R} & \otimes \\ \hline \mathbf{R} & .62 & .38 \\ \otimes & .15 & .85 \end{array}$$

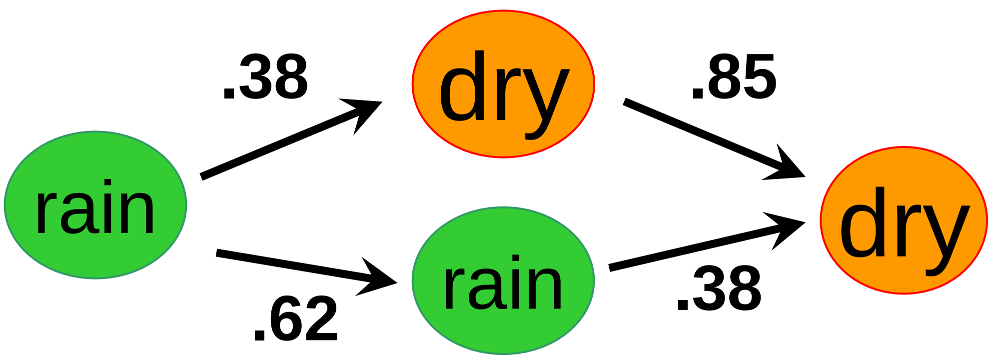
15% (11/72) probability of dry to rain

85% (61/72) probability of staying dry

$$\mathbf{R} = \text{rain} \qquad \otimes = \text{dry}$$

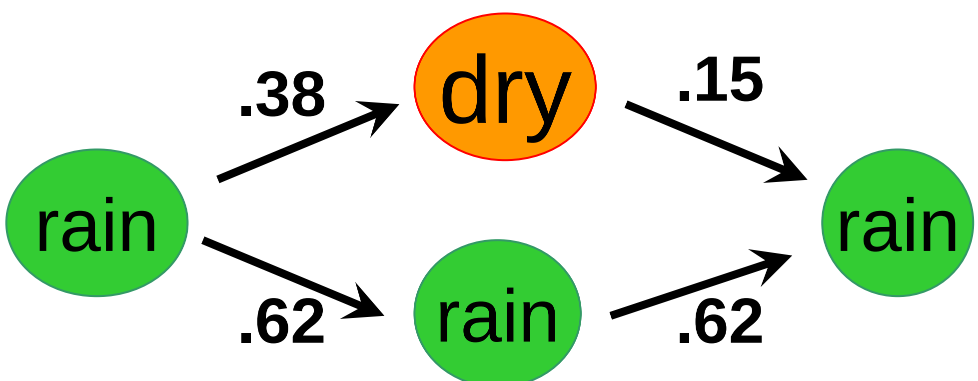
	R	⊗
R	.62	.38
⊗	.15	.85

What can we say about the 12 h transition? From 00-06 to 12-18?



$$0.38 \cdot 0.85 + 0.62 \cdot 0.38 = 0.56$$

$$0.38 \cdot 0.15 + 0.62 \cdot 0.62 = 0.44$$

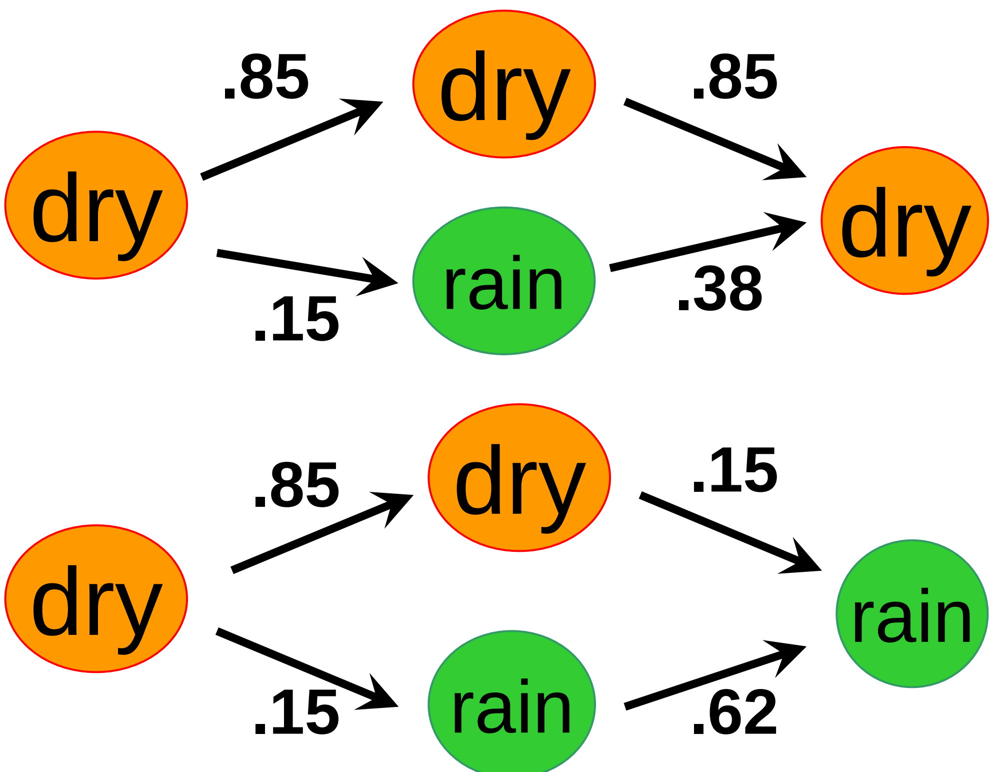


R	.44	.56
⊗	.	

$$\begin{array}{c} \\ R \\ \otimes \end{array} \begin{array}{cc} R & \otimes \\ \end{array}$$

$$\begin{array}{c|cc} & R & \otimes \\ \hline R & .62 & .38 \\ \otimes & .15 & .85 \end{array}$$

What can we say about the 12 h transition? From 00-06 to 12-18?



$0.85 \cdot 0.85 + 0.62 \cdot 0.38 = 0.78$

$0.85 \cdot 0.15 + 0.62 \cdot 0.62 = 0.22$

$$\begin{array}{c|cc} & R & \otimes \\ \hline R & .44 & .56 \\ \otimes & .22 & .78 \end{array}$$

The matrice can also be regarded as an **algebraic transition matrix**

$\mathbf{R} = \text{rain} \quad \otimes = \text{dry}$

Next 6 hours

Last 6 hours

$$\begin{array}{cc} & \begin{array}{cc} \mathbf{R} & \otimes \end{array} \\ \begin{array}{c} \mathrm{R} \\ \otimes \end{array} & \begin{pmatrix} .62 & .38 \\ .15 & .85 \end{pmatrix} \end{array}$$

Then the full force of **3000 years of algebra** can be applied, in particular matrix algebra

Some matrice algebra

A brief repetition of matrix algebra...

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \cdot 5 + 2 \cdot 6 \\ - \end{pmatrix} = \begin{pmatrix} 17 \\ - \end{pmatrix}$$

A brief repetition of matrix algebra...

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} 1 \cdot 3 + 2 \cdot 5 & - \end{pmatrix} = \begin{pmatrix} 13 & - \end{pmatrix}$$

A brief repetition of matrix algebra...

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} 1 \cdot 3 + 2 \cdot 5 & - \\ - & - \end{pmatrix} = \begin{pmatrix} 13 & - \\ - & - \end{pmatrix}$$

Matrix multiplication yields a forecast

"Forecast" 12-18 h later

$$\begin{pmatrix} .62 & .38 \\ .15 & .85 \end{pmatrix}^2 = \begin{pmatrix} .44 & .56 \\ .22 & .78 \end{pmatrix}$$

The matrix multiplication continues...

$$\begin{pmatrix} .62 & .38 \\ .15 & .85 \end{pmatrix}^4 = \begin{pmatrix} .33 & .67 \\ .26 & .74 \end{pmatrix}$$

After repeated multiplications the values converge towards the ”climate”

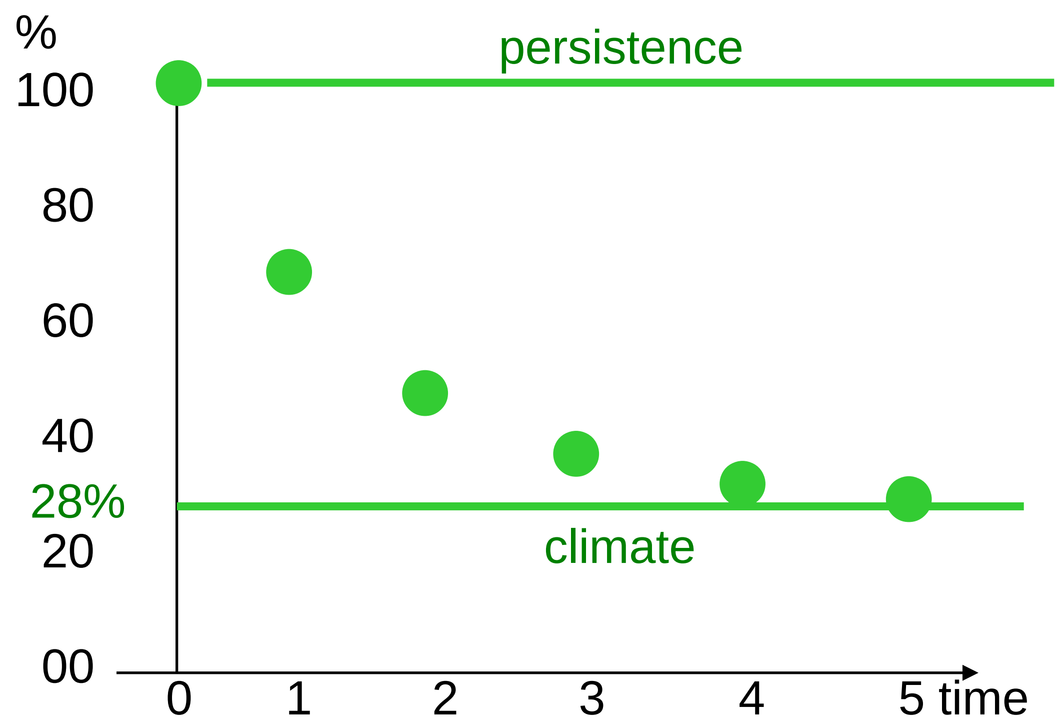
$$\begin{pmatrix} .62 & .38 \\ .15 & .85 \end{pmatrix}^8 = \begin{pmatrix} .72 & .28 \\ .72 & .28 \end{pmatrix}$$

Matrix = climate matrix + residual

Persistence — Variability

$$\begin{pmatrix} .85 & .15 \\ .38 & .62 \end{pmatrix}^N = \begin{pmatrix} .72 & .28 \\ .72 & .28 \end{pmatrix} + 0.47^N \begin{pmatrix} .28 & -.28 \\ -.72 & .72 \end{pmatrix}$$

$$\begin{pmatrix} .72 & .28 \\ .72 & .28 \end{pmatrix}^N = \begin{pmatrix} .72 & .28 \\ .72 & .28 \end{pmatrix}$$



Similarities with forecast models:

Obs/analysis → Model → Forecast

$$\begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}.62 & .38\\ .15 & .85\end{pmatrix} = \begin{pmatrix}.62 & .38\end{pmatrix}$$

Rain

…more examples:

Obs/analysis $\longrightarrow$ Model $\longrightarrow$ Forecast

$$\begin{pmatrix} .70 & .30 \end{pmatrix} \begin{pmatrix} .62 & .38 \\ .15 & .85 \end{pmatrix} \quad \begin{pmatrix} .48 & .52 \end{pmatrix}$$

Left eigenvector and -value

$$\begin{pmatrix} .28 & .72 \end{pmatrix} \begin{pmatrix} .62 & .38 \\ .15 & .85 \end{pmatrix} =$$

$$= \begin{pmatrix} .28 & .72 \end{pmatrix} = 1 \cdot \begin{pmatrix} .28 & .72 \end{pmatrix}$$

Right eigenvector and -value

$$\begin{pmatrix} .62 & .38 \\ .15 & .85 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1 \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

The initial transition matrix can be decomposed into a weighted sum of two new matrices

$$\begin{pmatrix} .62 & .38 \\ .15 & .85 \end{pmatrix}^{n} = \begin{pmatrix} .28 & .72 \\ .28 & .72 \end{pmatrix} + \underbrace{0.47^{n}}_{\text{Eigen value}} \cdot \begin{pmatrix} .72 & -.72 \\ -.28 & .28 \end{pmatrix}$$

Persistence probabilities | Climate probabilities | Time folding factor | Variability

Meteorological interpretation

END